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EVALUATION OF AERODYNAMIC COEFFICIENTS OF LONGITUDINAL SHORT-TIME MOVEMENT OF AIRCRAFT

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Abstract. *The method of evaluation of aerodynamic coefficients of longitudinal short-time movement of aircraft with small dimension of the basis is presented.*

Keywords: identification; aerodynamic coefficients; aircraft.

Introduction and Problem statement

Disturbed longitudinal movement of an aircraft describes by a simple model, which has small dimension of the basis and this fact partly eliminates the problem of impropriety solution of evaluation aerodynamic coefficients (ADC) task. Basic error of estimates is their displacement due to proximity of the linear model, which does not include the following factors:

1. Aircraft is asymmetric relatively the plane XOZ. Consequently, the dynamics of pitching-up and pitching-down, even for small $\Delta\alpha$, will be different

2. Under very small deviations of rudders $\Delta\delta_B$, pitch angle $\Delta\alpha$ and angular velocity angle $\Delta\bar{\omega}_z$ the linear model is accurate, but it is poorly identified because of increasing the ratio of "noise – signal" and impact is significant others, not calculated in the model, factors. The main factors $\Delta\alpha,\ \Delta\bar{\omega}_z,\ \Delta\delta_B$ have to be substantial, but they have limitations below - to materiality top - by nonlinearity.

3. With a slow change the dynamics is absent and three-dimensional basis $\Delta\alpha,\ \Delta\bar{\omega}_z,\ \Delta\delta_B$ degenerates into a one – or two-dimensional: the inverse problem is incorrect. In a very sudden change $\Delta\delta_B$, there is an additional "antidamping" by straightening slightly bent before axis X_1X_1 of aircraft. Then the total damping $\Delta\bar{\omega}_z$ in its evaluation on real data will be understated.

Therefore, from the physically realizable natural experiment is desirable to find estimations ADC, as accurately, as possible with constraints on time intervals of the experiment changing and amplitude deviations of variables. Recall, that the aerodynamic coefficient is derivative, for example, since pitch m_{z_1} for the respective variable $\alpha,\ \omega_{z_1}$.

Solution of problem

Reasonable compromise between the linear model and the nonlinear model is represented in the first and second terms of decomposition nonlinear model in a Taylor series:

$$
\begin{aligned}
m_{z_1}(t) &= m_{z_1}(t_0) + \left.\frac{\partial m_{z_1}}{\partial\alpha}\right|_{t_0}\Delta\alpha(t) + \left.\frac{\partial m_{z_1}}{\partial\bar{\omega}_{z_1}}\right|_{t_0}\Delta\bar{\omega}_{z_1}(t) + \\
&+ \left.\frac{\partial m_{z_1}}{\partial\delta_B}\right|_{t_0}\Delta\delta_B(t) + \frac{1}{2}\left.\frac{\partial^2 m_{z_1}}{\partial\alpha^2}\right|_{t_0}\left(\Delta\alpha^2(t)\right)\times \\
&\times\left[+\left.\frac{\partial^2 m_{z_1}}{\partial\bar{\omega}_{z_1}}\right|_{t_0}\left(\Delta\bar{\omega}_{z_1}(t)\right)^2 + \left.\frac{\partial^2 m_{z_1}}{\partial\delta_B}\right|_{t_0}\left(\Delta\delta_B(t)\right)^2\right] + \\
&+ \left.\frac{\partial^2 m_{z_1}}{\partial\alpha\partial\bar{\omega}_{z_1}}\right|_{t_0}\Delta\alpha(t)\Delta\bar{\omega}_{z_1}(t) + \left.\frac{\partial^2 m_{z_1}}{\partial\alpha\partial\delta_B}\right|_{t_0}\Delta\alpha(t)\times \\
&\times\Delta\delta_B(t) + \left.\frac{\partial^2 m_{z_1}}{\partial\bar{\omega}_{z_1}\partial\delta_B}\right|_{t_0}\Delta\bar{\omega}_{z_1}\Delta\delta_B(t)
\end{aligned}
\qquad (1)
$$

This model takes into account the asymmetrical modes pitch-up and pitch-down. But a large number of its members, mutually correlated, limitations and proximity of the measurements, limitation of the regime time changing the speed and height, make the task of evaluating all ADC incorrect. Accordingly the purpose of the experiment, we are interested only in the linear terms of decomposition (1).

Rewrite (1) as follows:

$$
\begin{aligned}
&\Delta m_{z_1}(t) = \\
&= \left[m_{z_1}^{\alpha} + \left(m_{z_1}^{\alpha^2}\Delta\alpha(t) + m_{z_1}^{\alpha\omega_я}\Delta\bar{\omega}_{z_1}(t) + m_{z_1}^{\alpha\delta_B}\Delta\delta_B(t)\right)\right]\Delta\alpha(t) + \\
&+ \left[m_{z_1}^{\bar{\omega}_{z1}} + \left(m_{z_1}^{\overline{\omega_{z1}}\alpha}\Delta\alpha(t) + m_{z}^{\bar{\omega}_{z1}^2}\Delta\bar{\omega}_{z_1}(t) + m_{z_1}^{\bar{\omega}_{z1}\cdot\delta_B}\Delta\delta_B(t)\right)\right]\Delta\bar{\omega}_{z_1}(t) + \\
&+ \left[m_{z_1}^{\delta_B} + \left(m_{z_1}^{\delta_B\alpha}\Delta\alpha(t) + m_{z}^{\delta_B\bar{\omega}_{z1}}\Delta\bar{\omega}_{z_1}(t) + m_{z_1}^{\delta_B^2}\Delta\delta_B(t)\right)\right]\Delta\delta_B(t),
\end{aligned}
\qquad (2)
$$

where each factor formed from the desired ADC and its linear dependence on $\Delta\alpha,\ \Delta\omega_z,\ \Delta\delta_B$. If on some approximation degree in the expressions for the coefficients of equation (2) variables $\Delta\alpha(t)$, $\Delta\omega_{z_1}(t),\ \Delta\delta_B(t)$ approximate by the corresponding step functions $\Delta\alpha(\infty)\cdot 1(t),\ \Delta\omega_{z_1}(\infty)\cdot 1(t)$, $\Delta\delta_B(\infty)\cdot 1(t)$, where argument (∞) denotes to the time of constant value appearance so, with accuracy to this approximation, equation (2) can be represented as follows:

$$\Delta m_{z_1}(t) \cong a_2 \Delta\alpha(t) + a_1 \Delta\omega_z(t) + в_1 \Delta\delta_B(t) \qquad (3)$$

where

$$a_1 = m_{z_1}^{\bar{\omega}_z} + \left[m_{z_1}^{\alpha\bar{\omega}_z} \Delta\alpha(\infty) + m_{z_1}^{\bar{\omega}_z^2} \Delta\bar{\omega}_{z_1}(\infty) + m_{z_1}^{\alpha\delta_B} \Delta\delta_B(\infty) \right],$$

$$a_2 = m_z^{\alpha} + \left[m_z^{\alpha^2} \Delta\alpha(\infty) + m_{z_1}^{\alpha\bar{\omega}_z} \Delta\bar{\omega}_{z_1}(\infty) + m_{z_1}^{\alpha\delta_B} \Delta\delta_B(\infty) \right],$$

$$в_1 = m_z^{\delta_B} + \left[m_z^{\delta_B\alpha} \Delta\alpha(\infty) + m_z^{\delta_B\bar{\omega}_z} \Delta\bar{\omega}_{z_1}(\infty) + m_z^{\delta_B^2} \Delta\delta_B(\infty) \right].$$

Changing the amplitude $\Delta\delta_B(\infty)$, we can proportionally change offset coefficients a_1, a_2, b_1 relatively unknown ADC: $m_{z_1}^{\bar{\omega}_z}$, $m_{z_1}^{\alpha}$, $m_z^{\delta_B}$.

Then stating the experiment will be on sequence of the "steps" (stairs) elevator with varying amplitude, determination of the shifted coefficients a_1, a_2, b_1 of equation (3) and elimination of this bias by linear approximation coefficients a_1, a_2, b_1 as a function of $\Delta\delta_B(\infty)$ or $\Delta\alpha(\infty)$, or $\Delta\omega_z(\infty)$.

$$\begin{aligned} a_{1i} &= m_{z_1}^{\bar{\omega}_z} + k_1 \Delta\alpha_i(\infty), \\ a_{2i} &= m_{z_1}^{\alpha} + k_2 \Delta\alpha_i(\infty), \\ в_{1i} &= m_z^{\delta_B} + k_3 \Delta\alpha_i(\infty), \end{aligned} \qquad (4)$$

where i – number of signals of varying amplitude.

Test example. Exact nonlinear model

$$y(k) = \sum_{j=1}^{3} x_j(k) + \sum_{j,q=1,\, j\geq q}^{3} x_j(k) x_q(k) \qquad (5)$$

with single coefficients β_i for samples of varying amplitude $x_{\max}$ approximated by its linear part for such signals

$$\begin{aligned} x_1(k) &= x_{\max}(l) \sin\left(\pi \frac{k-1}{M-1}\right), \\ x_2(k) &= x_{\max}(l) \sin\left(2\pi \frac{k-1}{M-1}\right), \\ x_3(k) &= x_{\max}(l) \cos\left(2\pi \frac{k-1}{M-1}\right), \end{aligned} \qquad (6)$$

where $k = \overline{1, M}$, $l = \overline{1, 4}$.

Estimates $\hat{\beta}_j$ $(j = 1, 2, 3)$ were calculated by the least squares method (LSM). Index $\varepsilon^T\varepsilon$ of error ε approximation $y(k)$ by the model $y(k) = \hat{\beta}_1 x_1(k) + \hat{\beta}_2 x_2(k) + \hat{\beta}_3 x_3(k)$ was two orders of magnitude less than similar rules $\|\Delta y\|$. That solves the problem of approximation on the sufficient quality level. But ratios $\hat{\beta}_i$ were significantly biased (fig. 1). Linear regression dependence shifts $\Delta\beta_j$ (j = 1, 2, 3) from $x_{\max}(l)$, (l = 1, 2, 3, 4), identical to zero amplitude $x_{\max}$ to zero; under evaluation $\hat{\beta}_j$ – to true $\beta_j = 1$.

With regularization according to Tikhonov of LSM-evaluation of complete model's coefficients (5) gave almost zero value of $\varepsilon^T\varepsilon$ index. But it is approached for the various a priori values of β_{apr}, and estimation of $\hat{\beta}$ is close to β_{apr}, not to real, equal to one.

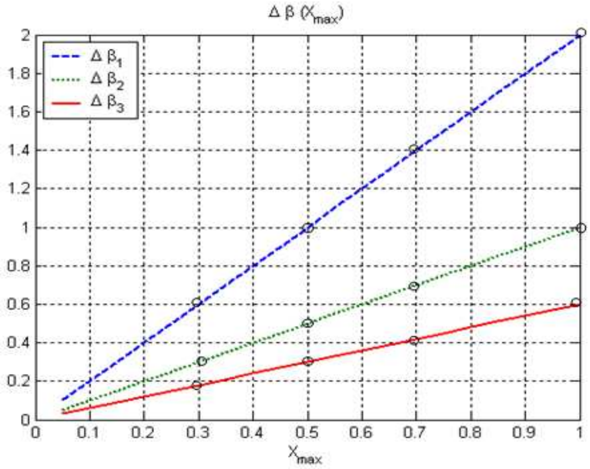

Fig. 1. Dependence $\Delta\beta(x_{\max})$

Real example. On fig. 2 there is a photo of seven mode changes in position of rudder $\delta_в$, angle of attack $\alpha(t)$ and angular velocity $\omega_{z_1}(t)$ of a light plane in short-periodic longitudinal motion. In each of seven regimes values of ADC deviated as a result of model's proximity (3) were determined.

According to them, reserve $\hat{\sigma}_n$ of aperiodic resistance on vertical overload was estimated, which was approximated by a linear dependence in function $\|\Delta\alpha\|$ (fig. 3).

$$\hat{\sigma}(\|\Delta\alpha\|) = 0,22 - 0,075\|\Delta\alpha\|. \qquad (7)$$

Unbiased estimate $\hat{\sigma}_n = 0,22$ is obtained by a linear approximation of the dependence (7) and calculate of its value at zero deviation. Averaging the results will give significantly underestimated value $\hat{\sigma}_n = 0,188$.

Further precision of $\hat{\sigma}_n$ can be achieved by fitting a regression estimates of ADC with smooth dependence on other flight parameters (speed, altitude, etc.).

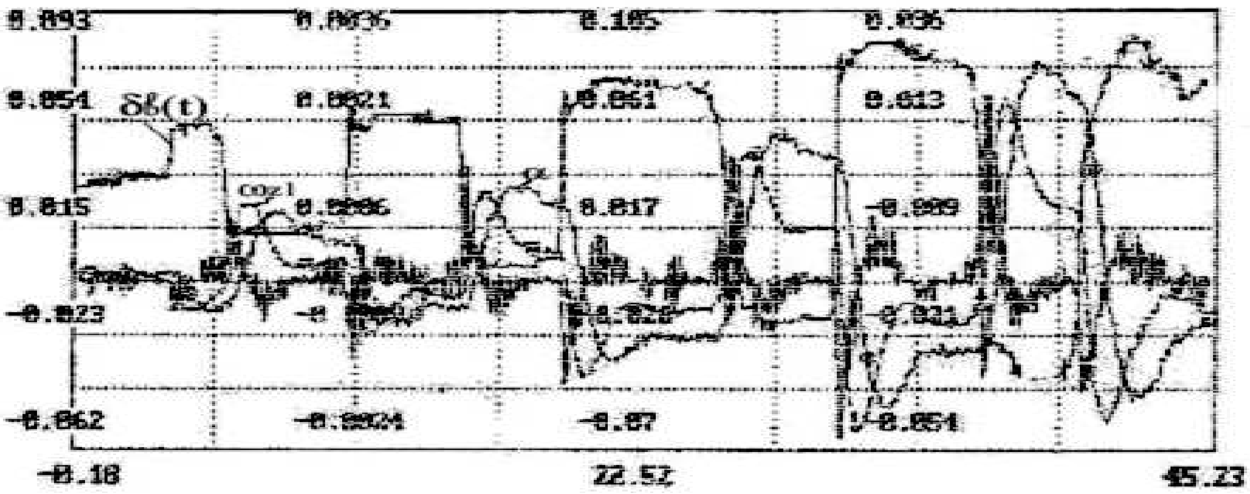

Fig. 2. Oscillogram of mode changes in rudder's position, angle of attack and angular velocity

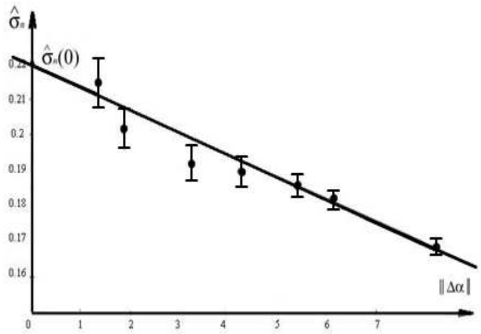

Fig. 3. Dependence of $\hat{\sigma}_n$ value on $\|\alpha\|$

Table provides a summary of the different types of aircraft data supporting the effectiveness of verification, the safety factor by fitting a linear regression function of various parameters of the flight. For various aircrafts dimension ΔX ranged from 2 to 6 and the number of modes from 15 to 190.

Comparison of the simple averaging with regression approximation σ_n from flight data

№ i/o	Aircraft type	RMSE %		Dimension of ΔX	Number of modes
		Models	Mean		
1	An-72	5	102	6	190
2	Il-86	7	31	2	25
3	Tu-154	4	13	4	70
4	Mig-29	7	50	4	50
5	M-17	0,5	1,5	2	15

Conclusions

For correctness of setting the task of ADC identification we should distinguish between the signal and the parametric approaches [1]. Their common objectives is to minimize $\varepsilon^T\varepsilon$, the difference – in the models (abstract and "physically" adequate) and the requirements for functional $\varepsilon^T\varepsilon$, as the evaluation function of ADC (or lax and strict convexity).

Unfortunately, in practice, the aircraft flight test is sometimes used to estimate the parameters of the signal identification, laying in the model is not a priori objective inaccurate (as calculated results or wind tunnel) coefficients and then "tuning" them from the minimum regularized functional. This delivers the apparent adequacy of the model object: error is small, close to the a priori estimates. Though they may differ from the actual physical parameters.

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А. М. Сільвестров. Оцінка аеродинамічних коефіцієнтів поздовжнього коротко-періодичного руху повітряних суден

Представлено метод оцінки аеродинамічних коефіцієнтів поздовжнього руху літака за короткий час з невеликим розміром основи.

Ключові слова: повітряні судна; короткочасний рух.

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А. Н. Сильвестров. Оценка аэродинамических коэффициентов продольного коротко-периодического движения воздушных судов

Представлен метод оценки аэродинамических коэффициентов продольного движения самолета за короткое время с небольшим размером основы.

Ключевые слова: воздушные суда; кратковременное движение.

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