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# COMBINED COMPLEMENTARY FILTER FOR INERTIAL NAVIGATION SYSTEM

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**Abstrakt.** *This paper presents analysis of the complementary algorithm for inertial navigation systems. Suboptimal algorithm for sensors data fusion of the navigation system is proposed. This method employs combined complementary filter approach and attitude error equation of the inertial navigation system. Simulation results are presented to demonstrate the performance of the proposed approach.*

**Keywords:** inertial navigation system, complementary filter, data fusion.

**Introduction.** The traditional approach to navigation systems employs Inertial Navigation System (INS) and Global Navigation Satellite System (GNSS), using different data fusion algorithms.

The majority of navigation data fusion algorithms are based on Kalman filter, particularly Extended Kalman filter (EKF). The EKF linearizes both the process and the observation functions with a first-order Taylor approximation. In practice, this approximation is the source of errors of the EKF [5], since the approximated none-linear problem is actually optimally solved by the corresponding linear Kalman filter. Besides it is difficult to know exact values for covariance matrix of noise rondom process. Together, these factors contribute to filter divergence.

In most cases leading researchers solve this problem using their own unique approaches. In particular in order to overcome divergence a lot of Kalman filter modifications were developed, e. g.: Yasvinsky algorithms, different robust [3] and adaptive extensions [4].

At present besides optimal state vector estimation (Kalman filter), there are other methods of data fusion, which are well proven in practice, e. g. in complementary filters. The feasibility of this method is provided by the fact that measurement of navigation data is based on different physical principles, and measurement errors remain in different frequency ranges.

Complementary filter approach allows to estimate only measured components of navigation system, in case of Kalman filter it is possible to estimate all components of state vector, in particular the angular orientation and sensor's instrumental errors (gyro and accelerometer biases).

**Complementary Filter.** This paper proposes the combined complementary filter: measured components of state vector estimated with complementary filter approach and indirectly measured attitude are extrapolated by means of INS error equation.

As data fusion of redundant navigation information, complementary filter is suggested, which is well known from Doppler inertial navigation systems (fig. 1).

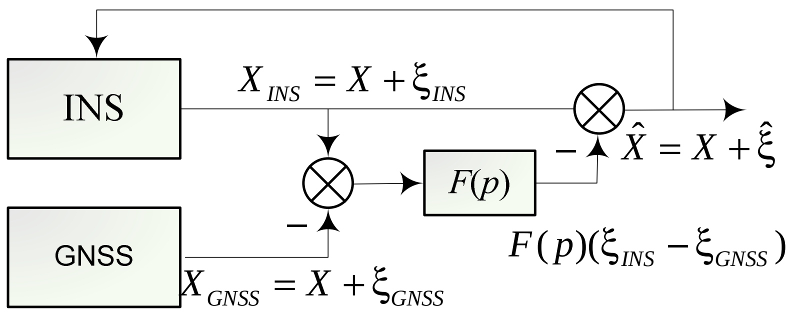


Fig. 1. Complementary filter

Instead of the classical aperiodic filter $F(p)$ in compensation schemes, authors propose to use a third order filter with variable structure [1].

$$F(p)=\begin{cases}\dfrac{1}{T_1 p+1} & \text{if} \quad t_{up}\le 3T_1;\\ \dfrac{3T_2 p+1}{(T_2 p+1)(T_2 p+1)(T_2 p+1)} & \text{if} \quad 3T_1<t_{up}\le 3T_2;\\ \dfrac{3T_3 p+1}{(T_3 p+1)(T_3 p+1)(T_3 p+1)} & \text{if} \quad 3T_2<t_{up}.\end{cases} \qquad (1)$$

Here $t_{up}$ is uptime of compensation complementary filter (1).

Studies have shown (fig. 2) that the results of such data fusion algorithm are not worse than of the Kalman filtering, without affecting the stability of estimation.

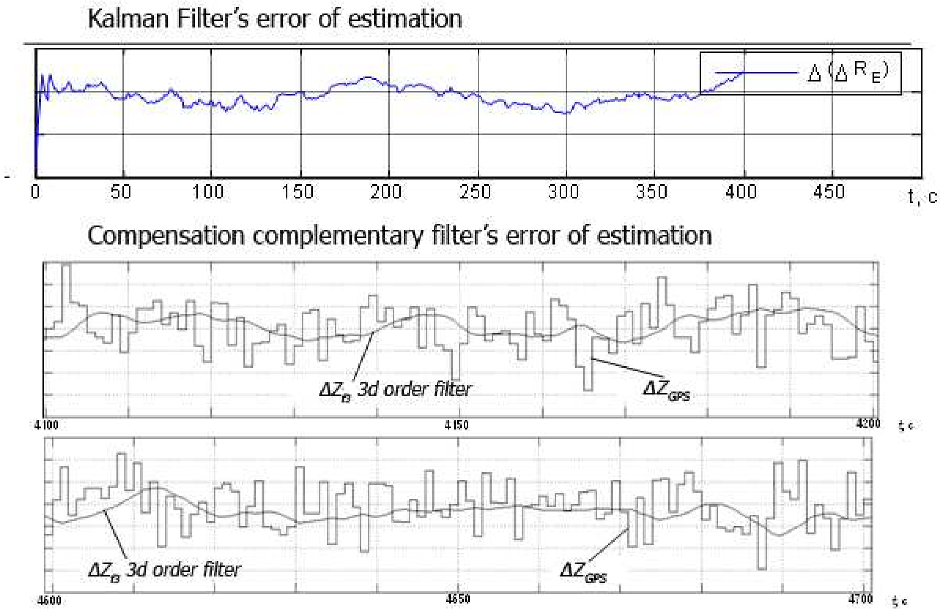


Fig. 2. Kalman and complementary filter performance

However, this algorithm does not estimate observable but not measurable state vector components, in particular angular orientation.

State vector of complex inertial satellite navigation system (ISNS) $x_{\text{ISNS},k}$ is based on the error equation of inertial and satellite systems $\mathbf{x}_{\text{ISNS},k}=\left|\mathbf{x}_{\text{INS},k} \quad \mathbf{x}_{\text{GNSS},k}\right|^T$, by using the optimal Kalman filter, the generalized state space equation of complex system errors can be written as:

$$\mathbf{x}_{\text{ISNS},k}=\mathbf{\Phi}_{\text{ISNS},k}\mathbf{x}_{\text{ISNS},k-1}+\xi_{\text{ISNS}} \qquad (2)$$

where $\mathbf{\Phi}_{\text{ISNS},k}=\begin{vmatrix}\mathbf{\Phi}_{\text{INS},k} & 0\\ 0 & \mathbf{\Phi}_{\text{GNSS},k}\end{vmatrix}$ is a known state propagation matrix, which is formed on the basis of the $\mathbf{\Phi}_{\text{ISNS}}$, $\mathbf{\Phi}_{\text{GNSS}}$ matrices, models of correlated components of GNSS and INS; $\xi_{\text{ISNS},k}=\left|\xi_{\text{INS},k} \quad \xi_{\text{GNSS},k}\right|$ is the vector of zero mean white Gaussian noise with covariance matrix $\mathbf{Q}_{\text{ISNS},k}$ (process noise matrix), of two corresponding navigation systems.

Equation for estimation of vector $\hat{\mathbf{x}}_{\text{ISNS},k}$, with certain assumptions, derived from the general equations of optimal filtering is as follows:

$$\hat{\mathbf{x}}_{\text{ISNS},k}=\tilde{\mathbf{x}}_{\text{ISNS},k|k-1}+\mathbf{K}_k(\mathbf{z}_{\text{GNSS},k}-\hat{\mathbf{z}}_{\text{ISNS},k}); \qquad (3)$$

$$\hat{\mathbf{z}}_{\text{ISNS},k} = \mathbf{G}(\mathbf{z}_{\text{INS},k} - \mathbf{M}_{\text{INS},k}\tilde{\mathbf{x}}_{\text{INS},k|k-1}) + \mathbf{M}_{\text{SNS},k}\tilde{\mathbf{x}}_{\text{GNSS},k|k-1}; \quad (4)$$

$$\tilde{\mathbf{x}}_{\text{ISNS},k|k-1} = \mathbf{\Phi}_{\text{ISNS},k}\tilde{\mathbf{x}}_{\text{ISNS},k-1}; \quad (5)$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^T\left(\mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + \mathbf{N}_k\right)^{-1}; \quad (6)$$

$$\mathbf{P}_{k|k-1} = \mathbf{\Phi}_{\text{ISNS},k}\mathbf{P}_{k-1}\mathbf{\Phi}_{\text{ISNS},k}^T + \mathbf{Q}_{\text{ISNS},k}; \quad (7)$$

$$\mathbf{P}_k = \mathbf{P}_{k|k-1} - \mathbf{K}_{f,k}\mathbf{H}_k\mathbf{P}_{k|k-1}; \quad (8)$$

$$\mathbf{H}_k = \mathbf{G}(\mathbf{z}_{\text{INS},k} - \mathbf{M}_{\text{INS},k}\tilde{\mathbf{x}}_{\text{INS},k|k-1}) + \mathbf{M}_{\text{GNSS},k}\tilde{\mathbf{x}}_{\text{GNSS},k|k-1}; \quad (9)$$

$$\mathbf{H}_k = \frac{\partial}{\partial V_{\text{ISNS},k}}\left[\mathbf{G}(\mathbf{z}_{\text{ISNS},k} - \mathbf{M}_{\text{INS},k}\dot{\mathbf{I}}_{\text{INS},k}) + \mathbf{M}_{\text{GNSS},k}\dot{\mathbf{I}}_{\text{GNSS},k}\right]$$

where $\mathbf{z}_{\text{GNSS}}$, $\mathbf{z}_{\text{INS}}$ are observation vectors of GNSS and INS; $\mathbf{G}$ is a known matrix of vector function $\mathbf{G}(\hat{\mathbf{x}}_{\text{ISNS},k})$, that connects the radio navigation signal parameters with the estimated state vector $\hat{\mathbf{x}}_{\text{ISNS},k}$; $\mathbf{M}_{\text{GNSS}}$, $\mathbf{M}_{\text{INS}}$ are the known matrices errors of the observation process from GNSS and INS; $\hat{\mathbf{z}}_{\text{ISNS}}$ is the estimation of an observation vector; $\tilde{\mathbf{x}}_{\text{ISNS}}$, $\tilde{\mathbf{x}}_{\text{INS}}$, $d\tilde{\mathbf{x}}_{\text{GNSS}}$ are errors of estimation of complex system, INS and GNSS errors; $\tilde{\mathbf{x}}_{\text{ISNS},k|k-1}$ и $\mathbf{P}_{k|k-1}$ are corresponding errors of INS, GNSS and covariance matrix for moment *k*, calculated based on measurements at the previous steps stamps *k*–1, *k*–2; **H** is a measurement matrix; **N** is a measurement noise covariance matrix.

Simulations of proposed filtering approach were done with simplified variant of inertial navigation system with the following equations of motion [1]:

$$\dot{\varphi} = \frac{V_N}{R_E + h}; \ \dot{h} = V_H; \ \dot{\vartheta} = \omega_z - \frac{V_N}{R_E + h};$$
$$\dot{V}_N = a_N - \frac{V_N}{R_E + h}V; \ \dot{V}_H = a_H + \frac{V_N}{R_E + h}V_N - g; \quad (10)$$
$$a_N = a_y\cos\vartheta - a_z\sin\vartheta; \ a_H = a_y\sin\vartheta + a_z\cos\vartheta.$$

$\varphi, h$ are latitude and height; $V_N, V_H$ are Northern and vertical components; $a_N, a_H$ are Northern and vertical components acceleration; $a_y, a_z$ are acceleration in body frame (accelero-meters output); $\vartheta$ is a pitch angle; $\omega$ is angular velocity in the body frame (gyro output); $R_E$ is Earth radius.

For simulation purposes next parameters of inertial sensors were used:

– gyro bias – $100^\circ / hr$;

– angular random walk – $1,2^\circ / \sqrt{hr}$;

– accelerometer bias – $10^{-2} g$;

– velocity random walk – $0,18 m/s/\sqrt{hr}$;

– Global Navigation Satellite System position precision – $7m(1\sigma)$;

– Global Navigation Satellite System velocity precision – $0,05 m/s(1\sigma)$.

Performance of complementary filter is presented on fig. 3 – 6. In particular fig. 6 shows the extrapolation of the INS angular orientation errors.

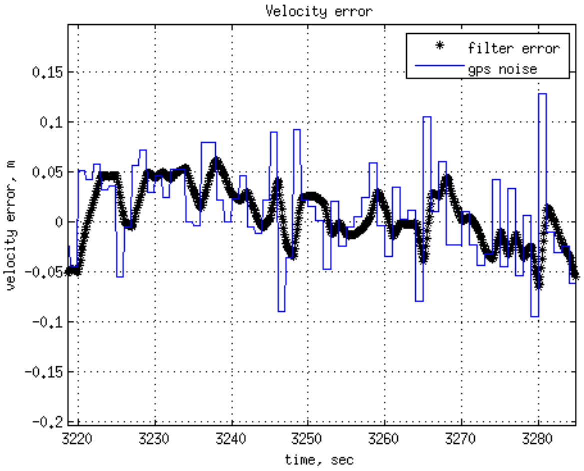


Fig. 3. Estimation error of velocity

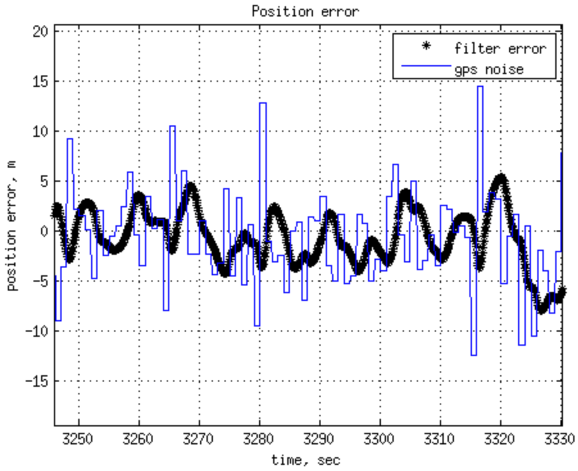


Fig. 4. Estimation error of position

**Attitude error extrapolator.** The following assumption was made in studies of the combined complementary filter: since the errors evolution of inertial navigation system can be observed from the output of complementary filter, it becomes possible to use this information to construct the attitude error extrapolator. Attitude errors can be described with the following equation:

$$\delta\dot{\vartheta} = -\frac{1}{R_E + h}\delta V_N + \frac{V_N}{(R_E + h)^2}\delta h + \varepsilon, \tag{11}$$

$\delta h$ is INS height error; $\delta V_N$ is INS North velocity error; $\delta\vartheta$ is attitude error; $\varepsilon$ is gyro bias.

This approach makes possible to predict the INS attitude error, like Kalman filter does on the prediction stage. In case of precise initial conditions, it is possible to get quite accurate predictions for the pitch error on the long time period, even for very noisy sensors. In our case, the angular random walk was chosen $1,2°/\sqrt{hr}$, for example inertial measurement unit Navchip ($ 1,000) from InterSense includes micromechanical gyroscopes with an angular random walk $0,18°/\sqrt{hr}$, and the device has one order of magnitude lower than the noise density.

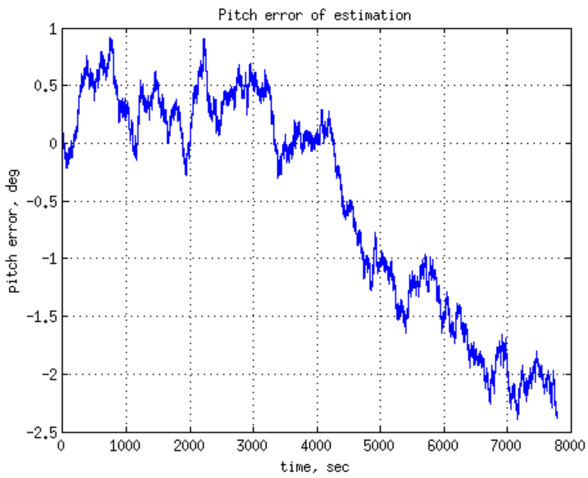


Fig. 5. Attitude estimation error

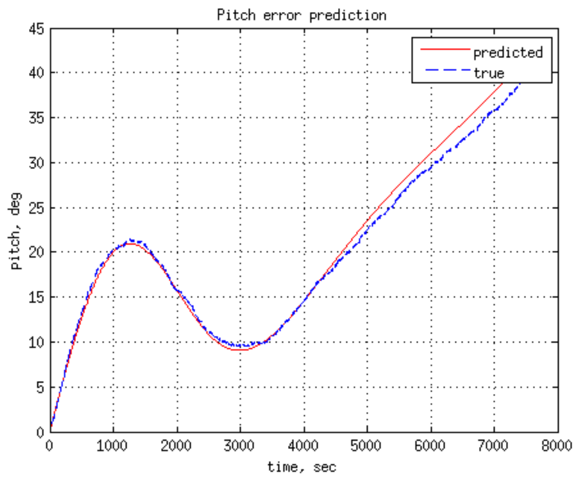


Fig. 6. Prediction of INS attitude error

This method has several drawback: large sensitivity to initial conditions of attitude INS errors. If gyro bias set with an accuracy of 10 %, the results of the extrapolation of attitude error significantly deteriorate (fig. 7).

Since there is no stage correction, extrapolated attitude INS error diverges with the time. The approach can be used quite successfully in case if unmanned air vehicles operation time or attitude correction time (for instance steady flight without acceleration) is less than diverge time of the filter.

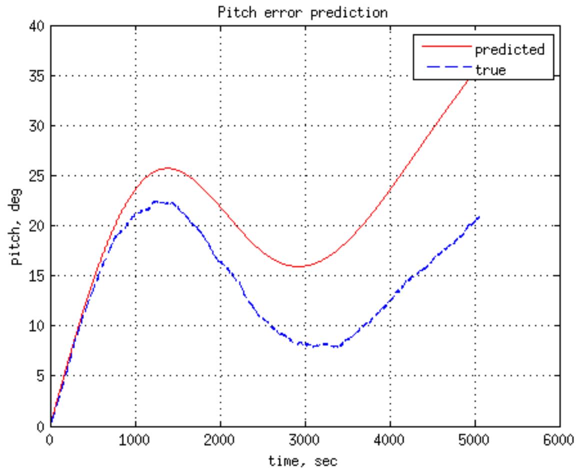


Fig. 7. Prediction of INS attitude error , in case of 10 % error in initial condition for gyro bias

**Conclusion.** The proposed approach to state estimation and data fusion for inertial and satellite navigation systems is faster and it robust to non-stationary random processes, e.g. sensor's scale factors or biases and on the other hand it can be quite easily implemented in the onboard digital computer.

## References

1. *Zakharin F. M., Sineglazov V. M., Filjashkin M. K.* 2011. Algorithmic support of inertial-navigation satellite systems. Kyiv, edition NAU. 320 p. (in Ukrainian).
2. *Clerk Maxwell J.* 1892. A Treatise on Electricity and Magnetism, 3rd ed., Vol. 2. Oxford: Clarendon. P. 68–73.
3. *Wan E. A. and R. Van Der Merwe*, 2000. «The unscented Kalman filter for nonlinear estimation», in Adaptive Systems for Signal Processing, Communications, and Control Symposium 2000. AS-SPCC. The IEEE 2000. P. 153–158.
4. *Mohamed A. H. and Schwarz K. P.* 1999. «Adaptive Kalman filtering for INS/GPS», Journal of Geodesy, Vol. 73. No.4. P. 193–203.
5. *Toledo-Moreo R., Gruyer D., and Lambert A.* 2011. "A theoretical analysis of the Extended Kalman Filter for data fusion in vehicular positioning," in ITS Telecommunications (ITST), 2011 11th International Conference on. P. 305–310.

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**Комбінований комплементарний фільтр для інерціальної навігаційної системи**

Розглянуто способи комплексної обробки інформації в інерціально-супутникових системах навігації. Під час побудови інтегрованих комплексів використовується комбінована схема фільтрації, яка об'єднує спосіб взаємної компенсації та елементи калмановської фільтрації. Наведено результати досліджень пропонованого алгоритму оцінки вектора стану.

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**Комбинированный комплементарный фильтр для инерциальной навигационной системы**

Рассмотрены способы комплексной обработки информации в инерциально-спутниковых системах навигации. При построении интегрированных комплексов используется комбинированная схема фильтрации, объединяющая способ взаимной компенсации и элементы калмановской фильтрации. Приведены результаты исследований предлагаемого алгоритма оценки вектора состояния.