UDC 681.518.2:621.391(045)

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ALGORIHTM OF STATE SPACE IDENTIFICATION OF AIRCRAFT MODEL

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This paper is devoted to the algorithm of state space identification of the flight dynamics models in the presence of sensor noise and biases. The goal of the identification procedure is not only the estimation of aircraft stability and control derivatives, but also the biases of sensors. It is achieved by using the procedure of the likelihood function minimization, based on the Kalman filter and the stochastic approximation procedure. The application technique of the least-squares method to a state space model in order to determine initial values of unknown parameters which are necessary to identify the state space model by maximum likelihood method is created. This algorithm was used for state space identification of the model of lateral-directional dynamics of small 6-seat aircraft.

Keywords: state space identification, aircraft model, sensor bias, least-squares method, maximum likelihood function.

Introduction. Identification of flight dynamics models on the basis of the flight test data in the presence of measurement noise and systematic errors (biases of the measuring systems and instruments) is a real problem especially for the small aircraft. For this class of aircraft it is impossible to apply the effective vibration insulation of the sensors. Its absence causes a high level of the measurement noise. Moreover, a lot of technical, economic and design restrictions exclude applying expensive sensors which have high accuracy; meanwhile the cheap and less precise ones have essential biases (systematic errors) in their output signals. Measurement noises and biases result the distorted parameter estimation of the dynamic model. Therefore, a minimization of the harmful effects of these factors at the first stage of the data processing is a very important task.

Parametrical identification methods of dynamics model presented by transfer functions or autoregression moving average equations are considered in many papers, e. g. in [1]. Since the aerodynamic characteristics of an aircraft are not the parameters of transfer functions and autoregression moving average equations but they are the stability and controllability derivatives of linearized state space model, it is better to perform the identification of flight dynamics models in state space.

Problem Statement. The structure of an aircraft dynamic model and records of input control signals $u(t)$ and responses $y(t)$ of the aircraft to them are the input data for the parametrical identification of flight dynamics model.

Mathematical model of the lateral and longitudinal motions of an aircraft is described by the linearized state space equations with constant parameters:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}; \\ \mathbf{y} &= \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} + \mathbf{b} + \boldsymbol{\xi}, \end{aligned} \tag{1}$$

where $\mathbf{A}$, $\mathbf{x}$ are the $n\times n$ state matrix and the $n\times 1$ state vector respectively; $\mathbf{B}$, $\mathbf{u}$ are the $n\times m$ control matrix and the $m\times 1$ control vector respectively; $\mathbf{y}$ is the $l\times 1$ measurement vector; $\mathbf{C}$ is the $l\times n$ observation matrix; $\mathbf{D}$ is the $l\times m$ matrix of the direct transfer from control input to output; $\mathbf{b}$ is the vector which elements are the systematic errors; $\boldsymbol{\xi}$ is the vector of Gaussian noise of measurement.

Having the records of input control signals and responses of the aircraft to them that contain useful component, measurement noise and systematic errors, it is necessary not only to determine a vector of parameters $\boldsymbol{\theta}$ of model (1), i. e. elements of matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$, but also the biases of sensors.

Problem Solution. As measurements of the state vector components are contaminated with the considerable noises it is desirable to use the maximum likelihood method (MLM) for the parametric identification of the aircraft state space model [2; 3]. This method results in the estimations, unbiased asymptotically, with the minimal variance in the case of Gaussian noise by selection of parametrical model which corresponds to the maximal value of likelihood function, that is

$$\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}\in\boldsymbol{\theta}_s} P(\mathbf{y}|\boldsymbol{\theta}).$$

where $\boldsymbol{\theta}, \hat{\boldsymbol{\theta}}$ are the vectors of unknown parameters and their estimation respectively; $P(\mathbf{y}|\boldsymbol{\theta})$ is the likelihood function. For convenience it is expedient not to find the maximum of likelihood function, but the minimum of the negative logarithm of this function [2], that is

$$J(\boldsymbol{\theta}) = -\ln P(\mathbf{y}|\boldsymbol{\theta}) = 0,5\{\sum_{i=1}^{N}(\mathbf{y}_i - \hat{\mathbf{y}}_i)^T \mathbf{R}_{in}^{-1}(\mathbf{y}_i - \hat{\mathbf{y}}_i) + N\ln|\mathbf{R}_{in}| + lN\ln(2\pi)\}, \quad (2)$$

where $\hat{\mathbf{y}}_i$ is the i-the estimation of the output vector of aircraft model; $(\mathbf{y}_i - \hat{\mathbf{y}}_i)$ is the i-the vector of innovations; $|\mathbf{R}_{in}|$ is the Frobenius norm of the innovation matrix; N is the number of measurement points (it depends on the length of realization); l is the size of the output vector $\hat{\mathbf{y}}$ (it depends on the number of measured values).

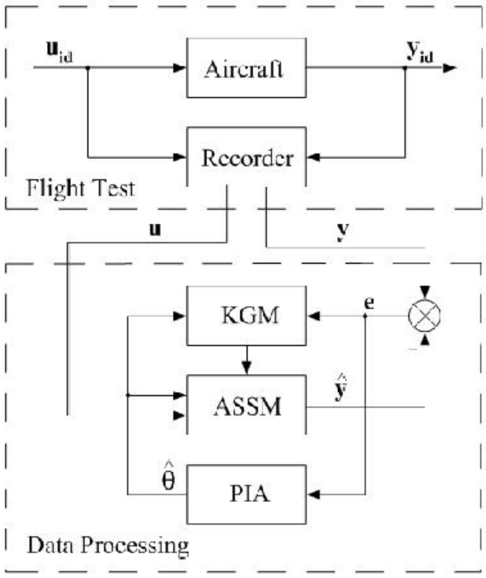

Fig. 1. The block diagram of the state space identification algorithm

The state space identification procedure of an aircraft on the basis of the maximum likelihood method is based on the application of the optimal Kalman observer of an aircraft dynamics and the optimization procedure together with the logarithmic likelihood function (2) as a cost function. The block diagram of this procedure is presented in fig. 1.

Digital data $\mathbf{u}$ and $\mathbf{y}$ received as a result of the flight test are processed in the block Data Processing. The Kalman gain matrix (KGM) and the aircraft state space model (ASSM) represent the optimum observer. The parametrical identification algorithm (PIA) represents the iterative procedure of the parametrical optimization which arranges the model parameters using the error $\mathbf{e}$.

Since it is necessary to estimate the parameters of an aircraft model (1) in the presence of measurement noise and sensor biases, in [4] it is proposed to extend the state space of this model in a way of inclusion in it augmented (“dummy”) variables [4] which are the biases of sensors:

$$\mathbf{b}_s = [b_{s1},\ b_{s2},\ \dots,\ b_{s\mu}]^T.$$

After extension of state space the input vector $\mathbf{u}_{ext}$, the state vector $\mathbf{x}_{ext}$ and the output (measurement) vector $\mathbf{y}_{ext}$ are the following:

$$\begin{aligned} \mathbf{u}_{ext} &= \mathbf{u} = [u_1,\ \dots,\ u_m]^T, \\ \mathbf{x}_{ext} &= [\mathbf{x},\ \mathbf{b}_s]^T = [x_1,\ \dots,\ x_n,\ b_{s1},\ \dots,\ b_{s\mu}]^T, \\ \mathbf{y}_{ext} &= \mathbf{y} = [y_1,\ \dots,\ y_l]^T. \end{aligned} \quad (3)$$

As a result of the extension (3) the system (1) has 2μ zero eigenvalues of Hamiltonian matrix associated with the Riccati equation for the observer. Solution of the optimum observer synthesis problem in the presence of singular Hamiltonian matrix having high order multiple zero eigenvalues is practically impossible. In this case it is appropriate to use the randomization approach to the "dummy" variables [4] after which Hamiltonian matrix with non-singular covariance matrix of the process noise is also non-singular and the optimum observer synthesis problem is solved with the help of standard algorithm based on the stationary Kalman filtration.

The application of stationary Kalman filtration algorithm to the "dummy" variables results their coarse estimation since the state and observation matrices determined by optimization procedure are still unobservable. For the improvement of the estimation of sensor biases it is possible to take advantage of their constancy [5] (during one flight at least). It is known, that algorithms of the stochastic approximation give asymptotically unbiased estimation of a mean value [6], therefore, it is expedient to use this property for the estimation of constant biases. Application of algorithms of the stochastic approximation helps to improve the state space estimation of the model (1) the some component of which are unknown sensor biases. Advantages of such combined state space estimation are noted in particular in [6]. In this connection it is proposed to use additional correction for state variables that concern to bias b_{sj}. This correction is determined by algorithm of accelerated Kesten stochastic approximation [7]:

$$\hat{\hat{x}}_{bsj}(i+1) = \hat{x}_{bsj}(i+1) + \gamma(i)\left(y_{bsj} - \hat{y}_{bsj} \right), \tag{4}$$

where $\hat{x}_{bsj}(i+1)$ is the estimation of the state variable, which concerns to j-the bias on i-the step, received after Kalman filtering; $\gamma(i)$ is the gain of the stochastic approximation on i-the step.

It is necessary to notice, that, if the estimation process is optimal, then the innovation vector $(\mathbf{y}-\hat{\mathbf{y}})_{i+1}$ in the steady-state mode should have properties of the white noise. If the correlation functions of all estimations of biases tend to delta-functions, it corroborates the efficiency of estimation procedure of sensor biases.

To determine the initial values of vector of the unknown parameters which are necessary to start the minimization procedure of negative logarithm of the maximum likelihood function (2) it is better to use the least-squares method (LSM) [8]. Before using LSM it is necessary to filter measurement noises with the help of digital physical unrealizable symmetrical non-recursive filter [8].

To apply the well-known LSM to the state space model (1) it is necessary to present it in the form of autoregression moving average

$$\mathbf{Y} = \mathbf{\Phi}\boldsymbol{\theta} + \mathbf{E} \tag{5}$$

where $\mathbf{Y}$ is the vector of output signals of model; $\mathbf{\Phi}$ is the identifier matrix; $\mathbf{E}$ is the error vector. For model (5) as a result of identification the vector of unknown parameters $\boldsymbol{\theta}$ is determined by the formula [1]:

$$\hat{\boldsymbol{\theta}}_{LSM} = \left(\mathbf{\Phi}^T\mathbf{\Phi}\right)^{-1}\mathbf{\Phi}^T\mathbf{Y}.$$

Generally, when the number of state variables is n, the number of input control signals is m, the number of measured signals is l, the vector $\mathbf{Y}$ is the following:

$$\mathbf{Y} = [y_1(i)\ \ y_1(i+1)\ \cdots\ y_1(N)\ \ y_2(i)\ \cdots\ \ y_2(N)\ \cdots\ y_l(i)\ \ y_l(i+1)\ \cdots\ y_l(N)]^T.$$

The size of vector $\mathbf{Y}$ in this case is $l(N-i+1)\times 1$. The vector of unknown parameters has the size $l(n+m)\times 1$ and it is

$$\hat{\boldsymbol{\theta}}_{LSM} = [a_{11}\ \ a_{12}\ \cdots\ a_{1n}\ \ b_{11}\ \ b_{12}\ \cdots\ b_{1m}\ \cdots\ a_{n1}\ \ a_{n2}\ \cdots\ a_{nn}\ \ b_{n1}\ \ b_{n2}\ \cdots\ b_{nm}]^T.$$

The identifier matrix for model (1) can be presented in the form of a block-diagonal matrix [8]

$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{\Phi}_0 & \mathbf{O} & \cdots & \mathbf{O} \\ \mathbf{O} & \mathbf{\Phi}_0 & \cdots & \mathbf{O} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{O} & \mathbf{O} & \cdots & \mathbf{\Phi}_0 \end{bmatrix} \tag{6}$$

where blocks $\mathbf{\Phi}_0$ look like: $\mathbf{\Phi}_0 = \begin{bmatrix} \mathbf{X}_0 & \mathbf{U}_0 \end{bmatrix}$,

$$\mathbf{X}_0 = \begin{bmatrix} x_1(i-1) & x_2(i-1) & \cdots & x_n(i-1) \\ x_1(i) & x_2(i) & \cdots & x_n(i) \\ \vdots & \vdots & & \vdots \\ x_1(N-1) & x_2(N-1) & \cdots & x_n(N-1) \end{bmatrix}; \qquad \mathbf{U}_0 = \begin{bmatrix} u_1(i) & u_2(i) & \cdots & u_m(i) \\ u_1(i+1) & u_2(i+1) & \cdots & u_m(i+1) \\ \vdots & \vdots & \vdots & \vdots \\ u_1(N) & u_2(N) & \cdots & u_m(N) \end{bmatrix};$$

$\mathbf{O}$ is the zero matrix which size is $(N-i+1)\times(n+m)$. The size of the identifier matrix (6) is $l(N-i+1)\times l(n+m)$.

Results of State Space Identification of Lateral Motion Model. The approach cited above has been applied to the solution of the state space identification problem of the lateral motion of the de Havilland aircraft DHC-2 “Beaver” which “benchmark” model is known [9]. Describing the lateral motion model of this aircraft by the state space equations (1) the state vector $\mathbf{x}$; the input vector $\mathbf{u}$ and the measurement vector $\mathbf{y}$ are the following:

$$\mathbf{x} = [p,\ r,\ \nu]^T; \qquad \mathbf{u} = [\delta\alpha,\ \delta r]^T; \qquad \mathbf{y} = [\dot{p},\ \dot{r},\ a_y,\ p,\ r]^T,$$

where p, r are roll and yaw rates, rad/s; ν is lateral velocity, m/s; $\delta\alpha$, δr are aileron and rudder deflections, rad; $\dot{p}$, $\dot{r}$ are roll and yaw accelerations, rad/s^2; a_y is lateral acceleration, m/s^2. The state space matrices for this model are the following:

$$A = \begin{bmatrix} L_p & L_r & L_\nu \\ N_p & N_r & N_\nu \\ Y_p & Y_r & Y_\nu \end{bmatrix}; \quad B = \begin{bmatrix} L_{\delta\alpha} & L_{\delta r} \\ N_{\delta\alpha} & N_{\delta r} \\ Y_{\delta\alpha} & Y_{\delta r} \end{bmatrix}; \quad C = \begin{bmatrix} L_p & L_r & L_\nu \\ N_p & N_r & N_\nu \\ Y_p & Y_r & Y_\nu \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}; \quad D = \begin{bmatrix} L_{\delta\alpha} & L_{\delta r} \\ N_{\delta\alpha} & N_{\delta r} \\ Y_{\delta\alpha} & Y_{\delta r} \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

where $L_{(\cdot)}$, $N_{(\cdot)}$ are roll and yaw moment derivatives; $Y_{(\cdot)}$ are lateral force derivatives.

The vector of unknown parameters looks like

$$\mathbf{\theta} = [L_p,\ L_r,\ L_\nu,\ N_p,\ N_r,\ N_\nu,\ Y_p, Y_r,\ Y_\nu, L_{\delta\alpha},\ L_{\delta r},\ N_{\delta\alpha},\ N_{\delta r}\ Y_{\delta\alpha},\ Y_{\delta r}]^T$$

and the vector of sensor biases $\mathbf{b}_s = [b_{sp}, b_{sr}, b_{sa_y}]^T$.

Results of state space identification of the vector of unknown parameters $\mathbf{\theta}$ by least squared and maximum likelihood methods are presented in Table 1.

Relative error of parameter estimation for the aircraft lateral motion is less then 5 % for twelve parameters and it is less than 25 % for other three. That coincides with the result of identifiability.

The values of sensor biases, estimated with the help of accelerated Kesten stochastic approximation algorithm (4), go to the initial values of these biases (Table 2). It corroborates the efficiency of application of this algorithm.

Table 1

Results of Parametrical Identification

Parameter	Nominal value	Estimated by LSM	Estimated by MLM	Relative error, %
L_p, s^{-1}	-5,820	-5,1641	-5,6372	3,14
L_r, s^{-1}	1,782	1,523	1,7620	1,12
L_v, $rad/(m \cdot s^2)$	-0,097	-0,0597	-0,0805	17,03
$L_{\delta\alpha}$, s^{-2}	-16,434	-13,999	-16,315	0,72
$L_{\delta r}$, s^{-2}	0,434	0,1965	0,4317	0,54
N_p, s^{-1}	-0,665	-0,5404	-0,6590	0,90
N_r, s^{-1}	-0,712	-0,7289	-0,7026	1,31
N_v, $rad/(m \cdot s^2)$	0,0084	-0,0244	0,0105	24,60
$N_{\delta\alpha}$, s^{-2}	-0,428	-0,0968	-0,4443	3,82
$N_{\delta r}$, s^{-2}	-2,824	-2,6483	-2,8026	0,76
Y_p, $m/(rad \cdot s)$	-0,278	-0,3623	-0,3261	17,31
Y_r, $m/(rad \cdot s)$	1,410	1,3460	1,3983	0,83
Y_v, s^{-1}	-0,180	-0,1229	-0,1792	0,44
$Y_{\delta\alpha}$, $m/(rad \cdot s^2)$	-0,447	-0.3498	-0.4250	4,91
$Y_{\delta r}$, $m/(rad \cdot s^2)$	2,657	2,2479	2,5665	3,41

Table 2

Results of sensor biases estimation

Bias	Nominal value	Estimated value	Relative error, %
b_{sp}, rad/s	0,0050	0,0047	6,00
b_{sr}, rad/s	0,0050	0,0049	2,00
b_{sa_y}, m/s^2	0,0850	0,0871	2,47

The processes of estimation of sensor biases (b_{sp} and b_{sr}) are presented in fig. 2.

Conclusions. The most effective method of the state space identification of flight dynamics models is the mlm. The state space identification algorithm of an aircraft on the basis of this method is based on application of the optimal kalman observer of aircraft dynamics and optimization procedure together with the logarithmic likelihood function as a cost function. It is offered to use the “dummy” variable randomization, if the hamiltonian matrix that is associated riccati equation, is singular; to use the accelerated Kesten stochastic approximation algorithm for more precise definition of sensor biases. The technique of application of LSM to a state space model to determine the initial values of unknown parameters, which are necessary to

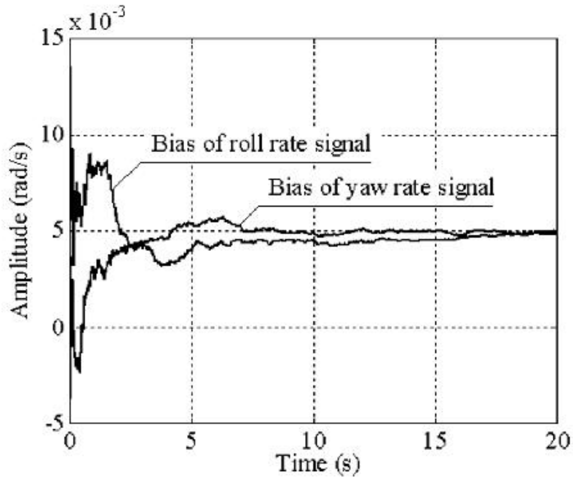

Fig. 2. The processes of sensor biases estimation

identify the state space model by MLM together with Kalman filter, is created. The efficiency of the proposed technique has been verified on the "benchmark" model of an aircraft lateral motion.

References

1. *Ljung L.* System Identification. Theory for the User. Prentice / L. Ljung. – Hall, Inc., 1987 p.
2. *Jategaonkar R.* Algorithms for aircraft parameter estimation accounting for process and measurement noise / R. Jategaonkar, E. Plaetschke // Journal of Aircraft. – 1989. – P. 360–372.
3. *Maine R.* Formulation and Implementation of a Practical Algorithm for Parameter Estimation with Process and Measurement Noise / R. Maine, K. Illif // Society of Industrial and Applied Mathematics, Journal of Applied Mathematics. – 1981. – Vol. 41, No. 3. – P. 558–579.
4. *Tunik A.* The Identification of the Flight Dynamics Models with Biased Sensors / A. Tunik, A. Klipa // Stability and Control Theory and Applications. – 2003. – Vol. 5, No. 1. – P. 41–48.
5. *Graupe D.* Identification of Systems / D. Graupe. – R. E. Krieger Publishing Company, 1976 p.
6. *Огарков М. А.* Методы статистического оценивания параметров случайных процессов / М. А. Огарков. – М.: Энергоатомиздат, 1990. – 207 с.
   [*Ogarkov M. A.* The Methods of the Statistical Estimation of the Random Process Parameters / M. A. Ogarkov. – Moscow, Energoatomizdat, 1990. – 207 p.] (In Russian).
7. *Kesten H.* Accelerated stochastic approximation / H. Kesten // Ann. Math. Stat.. – 1958. – No 29. – P. 41–59.
8. *Кліпа А. М.* Визначення початкових значень параметрів для ідентифікації моделі в просторі станів / А. М. Кліпа, А. А. Тунік // Електроніка та системи управління. – 2008. – №4(18). – С. 104–109.
   [*Klipa A. M.* Determinition of initial parameter values for identification of state space model / A. M. Klipa, A.A. Tunik // Electronics and Control Systems. – 2008. – No 4 (18), – P. 104–109.] (In Ukrainian).
9. *Rauw M.* The Flight Dynamics and Control Toolbox / M. Rauw. – MathWorks Company, 2000. – 263 p.

А. М. Кліпа

Алгоритм ідентифікації в просторі станів моделі літального апарата

Статтю присвячено розробці алгоритму ідентифікації в просторі станів моделей динаміки польоту за наявності шумів вимірювань та зміщень датчиків. Ціль розробленого алгоритму ідентифікації полягає не тільки в оцінюванні похідних стійкості та керування літального апарата, але й в оцінюванні змішень датчиків. Це досягається шляхом використання процедури мінімізації функції максимальної правдоподібності, заснованої на фільтрі Калмана і процедурі стохастичної апроксимації. Було запропоновано процедуру застосування методу найменших квадратів до моделі в просторі станів для визначення початкових значень невідомих параметрів, необхідних для ідентифікації моделі в просторі станів методом максимальної правдоподібності. Запропонований алгоритм було застосовано до моделі бічного руху легкого шестимісного літака.

А. Н. Клипа

Алгоритм идентификации в пространстве состояний модели летательного аппарата

Статья посвящена разработке алгоритма идентификации в пространстве состояний моделей динамики полета при наличии шумов измерения и смещений датчиков. Цель разработанного алгоритма идентификации состоит не только в оценивании производных устойчивости и управления летательного аппарата, но и в оценивании смещений датчиков. Это достигается путем использования процедуры минимизации функции максимального правдоподобия, основанной на фильтре Калмана и процедуре стохастической аппроксимации. Предложена процедура применения метода наименьших квадратов к модели в пространстве состояний для определения начальных значений неизвестных параметров, необходимых для идентификации модели в пространстве состояний методом максимального правдоподобия. Предложенный алгоритм был применен к модели бокового движения легкого шестиместного самолета.