UDC 625.735.(045)

[1]M. P. Mukhina,
[2]T. M. Stoyan

ANALYSIS OF MAGNETOMETER COMPONENTS ERRORS FOR CORRELATION-EXTREME NAVIGATION

Aerospace control systems institute, National Aviation University, Kyiv, e-mail:
[1]m_mukhina@inbox.ru, [2]siesta2010@ukr.net

Are the theory and techniques of navigation systems and guidance on geophysical fields (the so-called correlation-extreme navigation systems) considered. The classification of magnetic fields, on-board magnetometers, the main components of measurement errors of the Earth's magnetic field is described. Methods of modeling the errors components of three-component magnetometer are proposed.

Keywords: correlation-extreme navigation systems, three-component magnetometer, magnetic fields.

Actuality. Features of correlation-extreme navigation of the geophysical fields are the presence of certain anomalies or characteristic fields that are random functions of time and space. Navigation is performed by comparing the current realization of field with template map of field. The main criteria for comparison is the correlation function, which extreme (maximum value) coincides with the most probable location of the object on the map.

Magnetometers are distinguished for measuring the absolute values of the characteristics of the field and the relative changes in the field in space or in time. The last are called magnetic variometers. Magnetometers are classified by operating conditions (stationary, on movable platforms, etc.), and by the physical phenomena of their operations.

Therefore in navigation of flight the devices the physical fields of Earth can be divided into two classes:

1. Spatial fields of Earth which parameters are determined only in the definite point, – magnetic and gravitational fields.

2. Surface fields of Earth which parameters can be determined at earth surface, -relief field, thermal, optical, radiolocation contrast fields.

Spatial fields are defined in three-dimensional space; they are measured directly by magnetic and gravimetric equipment.

Another important factor is the presence of spatial fields over the whole Earth surface, while the surface fields that are available to be controlled from aircraft, are only over the land area of the Earth.

The Earth magnetic field (normal and anomalous together), due to magnetic rocks of the Earth's core and its layer of surface is characterized by the intensity and direction of the magnetic strength lines. On-board measurements of these fields as a module (absolute value of intensity), and as vector are done by the induction, quantum, ferro-probe magnetometers [1].

Problems statement. One of the key issues that determines the possibility of practical use of a physical field in navigational purposes, there is a question about map in formation of fields and the possibility of its updating. So there is a problem of the distribution of the components of geophysical fields through compliant processing under uncertainty and risk. In the classical approach of correlation-extreme navigation it is assumed the presence of a priori map information about geophysical fields, but the reliability of map is not absolute because anomalous constituents of majority of geophysical fields are slowly varying in time, in particular due to human activities (for example, anomalous components of the magnetic field may be changed due to the presence of large metal structures or availability of powerful sources of artificial electromagnetic radiation).

To provide high precision for determination of parameters of the magnetic field of the Earth on board it is necessary to solve the following tasks:

– creation of precision of measuring instruments onboard components and the full vector of the Earth's magnetic field (magnetometers);

– simulation of errors of magnetometers in order to get their statistic characteristics of field informativity [1].

The error of the magnetic field measurement is the result of the effect of all the factors that make a difference measurement of the magnetic field produced by the magnetometers from the a priori information about the anomalous magnetic field, which is contained in the memory of on-board digital computers. Let's assume the following four factors: $\sigma^2_{\Delta f\sim_1}$ *is an instrument errors of magnetometer;* $\sigma^2_{\Delta f\sim_2}$ *is the errors source of cartographic material and errors of the altitude magnetic maps;* $\sigma^2_{\Delta f\sim_3}$ *is a variation of the magnetic field ;* $\sigma^2_{\Delta f\sim_4}$ *is an uncompensated magnetic field of carrier* (fig. 1). Denote the mean square values of mentioned components accordingly. Let's assume all these disturbances are independent. That's why the total root mean square error is

$$\sigma_{\Delta f_\sim} = \sqrt{\sigma^2_{\Delta f\sim_1} + \cdots + \sigma^2_{\Delta f\sim_4}}. \qquad (1)$$

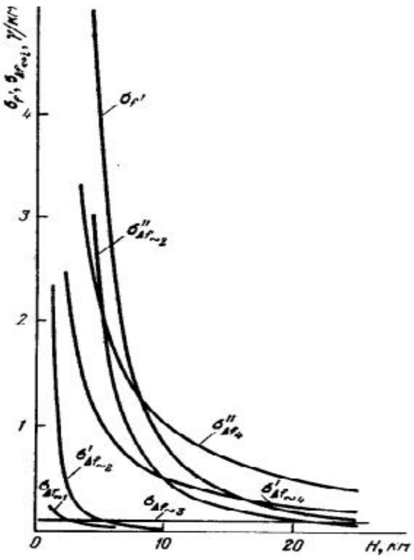

Fig. 1. Factors of errors component of magnetometer

Practical navigation for the anomalous magnetic field of the Earth can be made both by measurements of the field, and by the use of gradients (increments) of anomalies of the Earth magnetic field. Using the gradients allows us to eliminate the influence of permanent and slowly varying components of disturbances, which is essential when navigating by anomalies of the Earth magnetic field. Since the navigation by increment of anomalies of the Earth magnetic field will be considered, then so in formula (1) $\sigma^2_{\Delta f_\sim}, \sigma^2_{\Delta f\sim_1}, \cdots, \sigma^2_{\Delta f\sim_4}$ are variances of increments.

Analyses of errors component of magnetometer. *Root mean square error of gradient* $\sigma_{f'}$ depends on the altitude H of flight when approximation has form

$$\sigma_{f'} = \frac{A}{H^2},$$

where theoretical value of proportionality factor A is equal to

$$A = \frac{\sqrt{3}}{4\sqrt{2}} \sqrt{\sum_{i=1}^{N} \frac{k_i \sigma^2_{f0i}}{\alpha_i^2}},$$

where $k = 1{,}38 \cdot 10^{-23}\, J/K$ is Boltzmann constant; α – angle of deviation.

We can use altitude maps of gradient of anomalies of the Earth magnetic field to find proportionality factor A. In fig. 1 dependence $\sigma_{f'}(H)$ for $A = 80\gamma$ km is built. Let's estimate components (1) $\sigma^2_{\Delta f\sim_1}, \cdots, \sigma^2_{\Delta f\sim_4}$.

Instrumental errors of magnetometer. Gradient $\Delta f(x)$ of cross section of magnetic field $f(x)$ along flight path, calculated on the base Δl on the simple two-pointed scheme, equals to:

$$\Delta f(x) = \frac{f(x) - f(x - \Delta l)}{\Delta l}.$$

By raising the last expression to the second power

$$\Delta f^2(x) = \frac{f^2(x) - 2f(x)\,f(x-\Delta l) + f^2(x-\Delta l)}{\Delta l^2},$$

and by taking mathematical expectation of both parts, we obtain

$$\sigma^2_{\Delta f} = \frac{2\left[\sigma_f^2 - R_{ff}(\Delta l)\right]}{\Delta l^2}. \qquad (2)$$

On the base (2) for root mean square error of gradient calculation $\sigma^2_{\Delta f_{\sim 1}}$ is caused by instrumental error of magnetometer (the same as for the rest consisting errors). We will find

$$\sigma_{\Delta f_{\sim i}} = \frac{\sqrt{2[\sigma^2_{f_{\sim i}} - R_{ff_{\sim i}}(\Delta l)]}}{\Delta l},$$

where $\sigma_{f\sim i}$ and $R_{ff\sim i}$ – root mean square error and correlation function. We can assume that correlation radius of all disturbances less than base value Δl; then $R_{ff\sim}(\Delta l) = 0$ and $\sigma_{\Delta f\sim_i} = \frac{\sqrt{2}\sigma_{f\sim_i}}{\Delta l}$, in the following estimations we accept $\Delta l = 3\ H$, that corresponds to the value of correlation radius of anomalous magnetic field. Then

$$\sigma_{\Delta f_{\sim i}} = \frac{\sqrt{2}}{3}\frac{\sigma_{f_{\sim i}}}{H}. \qquad (5)$$

This equation will be used in the impact assessment of all consisting errors of gradient measurement of anomalies in Earth magnetic field, including errors of gradient measurement with the help of magnetometer

$$\sigma_{\Delta f_{\sim i}} = \frac{\sqrt{2}}{3}\frac{\sigma_{f_{\sim 1}}}{H},$$

assume that $\sigma_{f_{\sim 1}} = 0,5\gamma$ we will obtain $\sigma_{f_{\sim 1}} = \frac{0,24}{H}$.

Errors of the altitude magnetic maps. The errors influence of the original map material on the accuracy of altitude maps decreases with increasing altitude. If $\sigma_{f_{\sim 0}}$ is root mean square error of original map material and $\sigma_{f_{\sim H}}$ is root mean square error of altitude maps, so when sampling frequency Ω is chosen correctly, then

$$\frac{\sigma_{f_{\sim H}}}{\sigma_{f_{\sim 0}}} = \frac{1}{2\sqrt{2\pi}}\frac{1}{H\Omega}, \qquad \sigma_{\Delta f_{\sim 2}} = \frac{1}{6\sqrt{\pi}}\frac{\sigma_{f_{\sim 0}}}{\Omega H^2}.$$

In fig. 1 the dependence of error of making magnetic maps $\sigma''_{\Delta f_{\sim 2}}$ on altitude is shown for the sampling frequency $\Omega = 0{,}1$ km^{-1}. Comparison of graphics shows that accuracy of large-scale magnetic map is sufficient for navigation on the anomalies of the magnetic field. But when Ω decreases, the value $\sigma_{\Delta f_{\sim 2}}$ increases and errors of making the altitude magnetic maps become one of the main factors to determine the navigation accuracy of anomalies of the Earth magnetic field.

Impossibility of full compensation of the magnetic fields of the carrier is the main factor that limits the achievable navigation accuracy of anomalies of the Earth magnetic field. Even applying all necessary measures the root mean square error $\sigma_{f_{\sim 4}}$ of the residual deviation are units and tens. It leads to the error of gradients calculation. In fig. 1 graphics $\sigma'_{\Delta f_{\sim 4}}$ and $\sigma''_{\Delta f_{\sim 4}}$, are shown calculated according to $\sigma'_{\Delta f_{\sim 4}} = 20$. The graphics analysis resultsin fact that in the areas with are large-scale maps of anomalies of the Earth magnetic field, the achievable navigation accuracy is determined by the residual deviation of magnetic fields carriers.

Proposed measurement method. There are three widely used types of magnetometers: proton, quantum, ferro-probe. The first two types are used as absolute module measures of total vector of induction of magnetic field of Earth B_0, the third is used for measuring B_0 and its increments ΔB_0 and also to measure components of these values, their projections on the corresponding axis of measure device sensitivity $(B_i, \Delta B_0)$. Principle action of protonmagnetometer is based on the method of free nuclear induction based on internal atomic phenomenon – precession of nucleus of an atom around the force lines of magnetic field.

The quantum primary converter has the orientation dependence of signal value on the angle between the optical axis and the direction vector of the external magnetic field, which is expressed by the formula $A = A_{\max} \sin\alpha \cos\alpha$, where A – maximum magnitude of signal.

Principle of measurement induction of magnetic field with the help of ferro-probe magnetometers is based on the permalloy permeability that is an induction function of the magnetic field. This dependence is very sharp, and permalloy rod (probe) can be easily transferred from an unsaturated state, when it “concentrates” power line of the Earth magnetic field, in the saturated state, when it behaves almost like a magnetic body.

Technical characteristics of magnetometer. LSM303DLM module of geomagnetic sensors: includes three-component sensor of acceleration and three-component magnetometer. The technical characteristics are described in the table.

Technical characteristics of magnetometer

Main parameters		Features
Acceleration (max), ± g	8	– Analog supply voltage: 2,16 V to 3,6 V; – Digital supply voltage IOs: 1,8 V; – Power-down mode; – three magnetic field channels and three acceleration channels; – ±1,3 to ±8,1 gauss magnetic field full-scale; – ± 2 *g*/ ± 4 *g*/ ± 8 *g* dynamically selectable full scale; – High performance g-sensor; – I2C serial interface; – two independent programmable interrupt generators for free-fall and motion detection; – Accelerometer sleep-to-wake up function; – 6D orientation detection; – ECOPACK®, RoHS, and “Green” compliant.
Axes	*X*, *Y*, *Z*	
Sensitivity, LSB/g	0,256	
Frequency of cut-off, Hz	400000	
V_{CC}, B	from 1,71 to 3,7	
I_{CC}, mA	0,36	
T_A,°C	from –40 to 85	

Review of existing methods. Depending on the coordinate system the aeromagnetic measurements can be classified as:

1. In the geographical coordinate system measurements are provided by space stabilization of magnetosensitive block, which consists of three mutually orthogonal sensitive elements of ferroprobe.

2. Measurements of the body-fixed coordinate system where magnetosensitive block is rigidly fixed on the aircraft (fig. 2).

3. Combined method during with measurements in stabilized imaginary coordinate system, for instance, only in horizon plane.

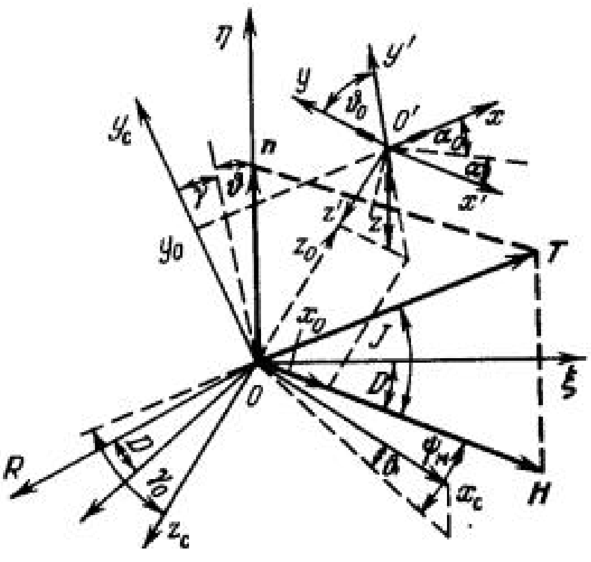

Fig. 2. Body-fixed coordinate system

The more widely spread method of measurement of magnetic field of Earth is by magneto-meter of the second type. Coordinate systems and angle designations are introduced in fig. 2 (T – total vector of magnetic field strength; n – vertical component of magnetic field of Earth; H – horizontal component of magnetic field of Earth; ηHR – magnetic coordinate system; J – magnetic declination; γ – bank angle; ψ_m – magnetic heading of aircraft; ϑ -pitch angle; $x_c y_c z_c$ – coordinate system connected with aircraft). Besides instrumental errors determined by the angles $\vartheta_0, \delta_0, \alpha_0$, errors of information about vertical $\Delta\vartheta'$ an $\Delta\gamma'$, errors are caused by inrigidity construction γ' and ϑ' which influence on the accuracy of measuring component of the Earth magnetic field can be determined with the help of corresponding operations with elements of matrix.

In the chosen coordinate system (see fig. 2) the projections of magnetic field of Earth on the axis of aircraft can be written as

$$AT_E = T_{cE},$$

where A – is a transform matrix from horizontal coordinate system into the body-fixed system; $T_E = [\xi\eta\zeta]^T$ – is a vector of the Earth magnetic field in the horizontal coordinate system, which components equal: $\xi = |T_E| \cos J \cos D$, $\eta = |T_E| \sin J$, $\xi = T_E \,|\cos J \sin D|$; $T_{cE} = [T_x T_y T_z]^T$ – is a vector of the Earth magnetic field in the body-fixed coordinate system; $|T_E|$ – is a module of the Earth magnetic field.

Equation of the inductive errors can be written in the form of Poisson’s equation in the matrix form:

$$\Delta T_{ci} = BT_{cE} = BAT_E,$$

where $\Delta T_{ci} = [\Delta T_{xi} \Delta T_{yi} \Delta T_{zi}]^T$ is a vector of disturbance from inductive magnetization in the body-fixed coordinate system, $B = \begin{bmatrix} a & b & c \\ d & e & f \\ k & g & h \end{bmatrix}$ – is a matrix of Poisson’s coefficients which characterizes inductive magnetization.

Measurements of the second method are more simple in the construction and they provide ability to compensate magnetic errors by the known methods, but they need to perform certain computing operations. To perform the second method of measurements we need compromise solution when we select place to install magnetosensitive block (provides minimum level of errors and significant constructive rigidity with respect to installation place of information source about vertical).

Selection of method depends on the definite problems, accuracy requirements, type of aircraft, technical characteristics etc.

Simulation model of magnetometer errors in Matlab. We have block that represented as magnetometer (fig. 3). Using the information about the state of magnetic field (x, y, z) it is necessary to obtain projections on the magnetic axis (T_m_x, T_m_y, T_m_z) and Wx, Wy, Wz are components of angular velocities.

In Matlab Euler angle direction cosine matrix with information about velocities of rotation of object (Wx, Wy, Wz) is simulated, where there are initial values (gam_0, tet_0, PSI_02) and ideal values (gam, tet, psi).

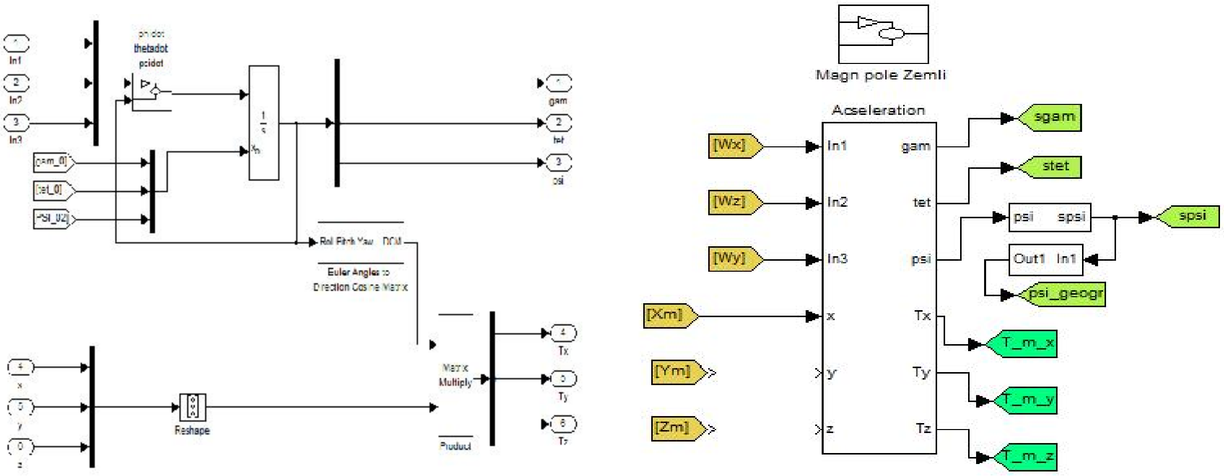

Fig. 3. Simulation model of magnetometer errors

The initial values of bank (gam), pitch (tet), yaw (psi) are formed, then the velocities of rotation of magnetic system in the space are formed.

Psi_m is the module used to simulate the magnetic heading (fig. 4).

Information about bank and pitch is given by INS. Below in fig. 5 Matlab simulation is represented that shows how bank and pitch angles change at the level flight.

Magnetometer heading differs from geographical in 0,6 deg with magnetic deviation *Dm* = 5.

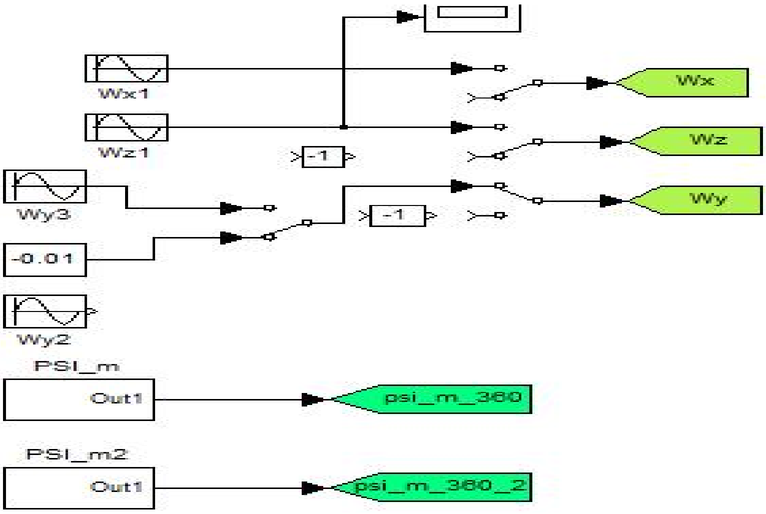

Fig. 4. Geographical and given magnetic heading

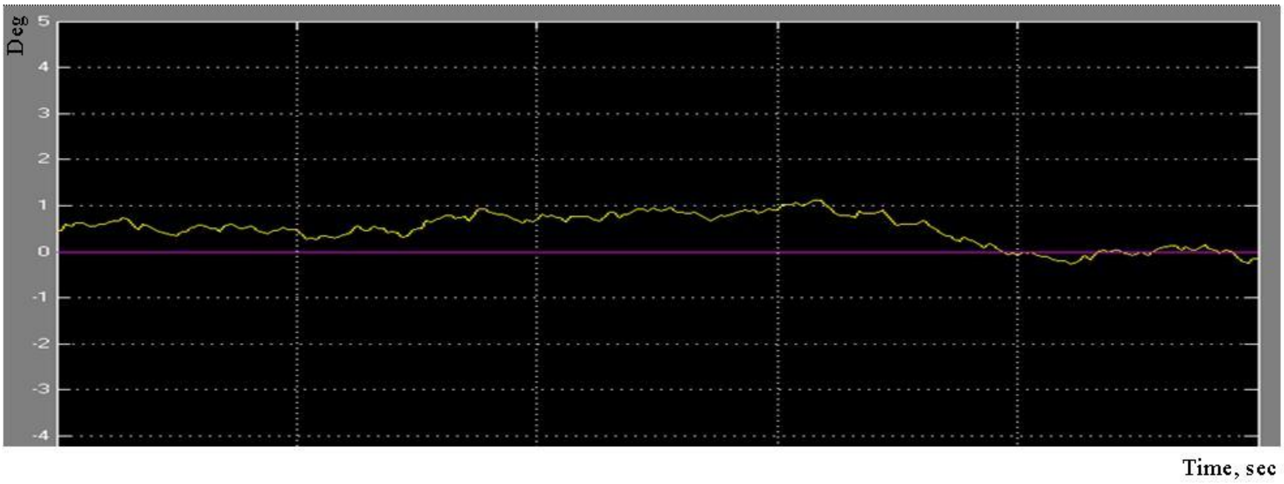

Fig. 5. Simulation of changes in bank and pitch angles with given heading

Conclusions. Measured results show errors deviation of bank and pitch angles in 0,6 deg. Proposed method allows us to investigate variations of the magnetic field at the area $300\times300\ \mathrm{km}^2$. These parameters correspond to stabilization parameters of gradient of the Earth magnetic field on the real data which were investigated in 1998 and 2010 in the same place. We can see that navigation problem of aircraft with using measurements of gradient of the Earth magnetic field can be solved with the help of modern devices, mainly three-component magnetometer. The described algorithm has certain limitations that affect its application. First, it is assumed that the size of a reference area larger than the possible positional errors. Otherwise, the real trajectory of the aircraft can be held outside the correction, and none of the stages of aggregation, as described above, will not work. Second, it is required that reference area contains anomaly gradient of about $0{,}1\ \mathrm{nTm}^{-1}$ or more, the amplitude is substantially higher than the stability of the gradient field. Third, the altitude above the surface of the Earth should not be more than $1\ \mathrm{km}$, because the amplitude of the gradient anomalies with altitude decreases much faster than the amplitude of the magnetic field anomalies. At altitudes more than 1 km, even in areas with intense magnetic anomalies of the gradient value has the order of $0{,}01\ \mathrm{nTm}^{-1}$.

References

1. *An Overview* of the Earth’s Magnetic Field. www.geomag.bgs.ac.uk/earthmag.html.
2. Белоглазов И. Н. Основы навигации по геофизическим полям / И. Н. Белоглазов, Г. И. Джанджгава, Г. П. Чигин. М.: Наука, 1985. – 328 с.
   [*Beloglazov I. N.* Navigation principles of geophysical fields. / I. N. Beloglazov, G. I. Dzhangdzhgava, G. P. Chigin. M.: Nauka, 1985. – 328 p.] (In Russian).
3. *Белоглазов И. Н.* Корреляционно-экстремальные системы / И. Н. Белоглазов, В. П. Тарасенко. – М.: Сов. радио, 1974. – 392 с.
   [*Beloglazov I. N.* Correlation-extreme systems / I. N. Beloglazov, V. P. Tarasenko. 1974 – 392 p.] (In Russian).
4. *Прецизионный* трехкомпонентный магнітометр / Режим доступа к электронному ресурсу: http://www.findpatent.ru/patent/246 /2467341.html
   [*Precision* three component magnetometer / Mode of access to the electronic resource: http://www.findpatent.ru/patent/246 /2467341.html] (In Russian).
5. *А. с. N 1633930*, СССР кл. G 01 C 17/38, 1989 Способ определения девиации курсоуказателя подвижного объекта / Режим доступа к электронному ресурсу: www.findpatent.ru/patent/163/1633930.html
   [*Author’s certificate N 1633930*, USSR cl. G 01 C 17/38, 1989 The method for determining the deviation course detector movable object / Mode of access to the electronic resource: www.findpatent.ru/patent/163/1633930.html] (In Russian).

М. П. Мухіна, Т. М. Стоян

Аналіз складових похибок магнітометра для задач кореляційно-екстремальної навігації

Розглянуто теорію і техніку систем навігації наведення по геофізичним полям (так званих кореляційно-екстремальних навігаційних систем). Досліджено класифікації магнітних полей, бортові магнітометри, основні складові похибок вимірювання магнітного поля Землі. Запропоновано методи моделювання складових похибок трикомпонентного магнітометра.

М. П. Мухина, Т. М. Стоян

Анализ составляющих погрешностей магнитометра для задач корреляционно-экстремальной навигации

Рассмотрены теория и техника систем навигации наведения по геофизическим полям (так называемых корреляционно-экстремальных навигационных систем). Исследованы классификации магнитных полей, бортовые магнитометры, основные составляющие погрешностей измерения магнитного поля Земли. Предложены методы моделирования составляющих погрешностей трехкомпонентного магнитометра.