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NON-LINEAR DYNAMICS OF ADVANCED AIRFRAME AIRCRAFT BASED ON STATE-TRANSFORMATION METHOD

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This paper focuses on nonlinear dynamics, optimization and control of longitudinal and lateral dynamics of flight vehicles. For advanced airframes and multiple control surfaces, it is impossible to linearize and decouple longitudinal and lateral dynamics. Nonlinear analysis and control are performed. For a given enabling airframe and control schemes, a robust design concept is researched. A consistent design is applied with a minimum level of simplifications and assumptions. Our findings are verified and demonstrated for practical problems. The reported design enables near-real-time implementation due to conceptual consistency, robustness, computational efficiency and algorithmic effectiveness. The proposed conceptis effective in design of flight control and management systems.

Keywords: tracking control, nonlinear aircraft dynamics, state-transformation method, proportional-integral control.

Introduction. In advanced fix- and variable-geometry wing aircraft and flight vehicles, various airframes and control solutions are deployed. For different aircraft classes (bombers, strike and multirole fighters, ground-attack aircraft and other), the mission-specific objectives are distinct. The overall goal is to guarantee a combat air superiority, supremacy and dominance. There is a broad spectrum of performance requirements and capability specifications which should be achieved.Advanced airframes, control surfaces and gas turbines must guarantee a broad range of conflicting specifications in expanded flight envelopes. The common specifications are agility, maneuverability, controllability, stability, speed,range, climb and turn rates, as well as other flying and handling qualities. These qualities and quantitative performance metrics must be achieved despite a low-signature airframe and control surfaces shaping, limits on most advanced structural materials, etc. Unconventional control, thrust-vectoring and other schemes may enable performance and capabilities of flight vehicles.

For conventional airframes, linear control theory was applied. The flight dynamics of advanced flight vehicles is open-loop unstable, and, described by highly-nonlinear differential equations. The longitudinal and lateral flight dynamics cannot be simplified or decoupled in expanded flight envelopes. One may be unable to apply linear control schemes which were used for simple airframes. Linear designs may suit partial flight envelopes and could lead to erroneous results in realistic flight envelopes under various engagement scenarios and flight conditions. Aircraft must ensure optimal achievable mission-specific flying and handling capabilities which are assessed by agility, controllability, maneuverability, stability and other performance characteristics, estimates and measures.

Aircraft's performance and capabilities can be enabled by flight control systems. This task implies multi-objective optimization and control for a given airframe and control schemes. Consistent, coherent and cohesive methods must be applied with a minimum level of simplifications and assumptions, despite potential design complexity.

Open-loop unstable flight vehicle dynamics is highly nonlinear. Model reductions, linearization, decoupling and decentralization cannot be applied in expanded flight envelopes. We apply and use a nonlinear model to design tracking control laws using the state transformation method. The control laws ensure near-optimal longitudinal and lateral dynamics. The flight- and mission-relevant performance functionals can be minimized using the design-specific performance integrands. We examine the role of nonlinearities, control bounds and uncertainties. Near-real-time design, adaptation and reconfiguration can be ensured. These features are of a particular importance to potentially accommodate control surface or airframe damages and failures. Adaptation and reconfiguration can be achieved in realistic flight scenarios and close-in high-g engagements if the vehicle remains to be controllable and stabiliazable. We examine descriptive flight conditions, envelopes and flight scenarios. Nonlinear simulations and data-intensive analysis are performed. Our findings and quantitative analyses are reported in details substantiating our design concept.

Nonlinear aircraft dynamics. Using the first principles of mechanics, applying linear and angular momentum, the nonlinear equations of motion are found within aircraft principal axes [1–3]. The rigid-body governing nonlinear equations for a twin-tail fighter are derived by using conventional notations and variables. The state and control variables are

$$x^{sys} = [v \ \alpha \ q \ \theta \ \beta \ p \ r \ \phi \ \psi]^T \text{ and } u = [\delta_{HR}\delta_{HL}\delta_{FR}\delta_{FL}\delta_C\delta_R]^T,$$

where v is the forward velocity; α is the angle-of-attack; q, p, and r are the pitch, roll and yaw rates; θ, φ and ψ are the pitch, roll and yaw angles; β is the sideslip angle; δ_{HR}, and δ_{HL} are the deflections of the right and left horizontal stabilizers; δ_{FR} and δ_{FL} are the deflections of the right and left flaps; δ_C is the canard deflection; δ_R is the rudder deflection.

The nonlinear equations of motion commonly used are[1 – 3]

$$\dot{x}^{sys}(t) = \boldsymbol{A}x^{sys} + \boldsymbol{F}(x^{sys}) + \boldsymbol{B}u, \ u_{\min} \le u \le u_{\max}, \tag{1}$$

$$x^{sys} \in X \subset \mathbb{R}^{c}, \ u \in U \subset \mathbb{R}^{m}, \ U = \{u \in \mathbb{R}^{m} \big| u_{\min} \le u \le u_{\max}, u_{\max} > 0, u_{\min} < 0\}.$$

The system nonlinearities are *mapped* by $\boldsymbol{F}(x^{sys})$. The system variables evolve within the flight envelope defined by X. There are mechanical limits on the deflection of control surfaces. The states and controls evolve in X and U.

The parameters significantly vary in the flight envelope. There are bounded uncertainties and perturbations of different origins. Using bounded

– parameter uncertainties p (parameter variations, unmodeled *fast* dynamics, unsteady aerodynamics, etc.);

– perturbations d (parameter perturbations, failures, damages, disturbances, etc.), one obtains the following system description

$$\dot{x}^{sys}(t) = \boldsymbol{A}(t,p)x^{sys} + \boldsymbol{F}(t, x^{sys}, d) + \boldsymbol{B}(t,p)u, x^{sys} \in X, \ u \in U, \ p \in P, \ d \in D. \tag{2}$$

The parameters can be identified in a near-real-time [4 – 6]. Correspondingly, in the analysis and design, we may use

$$\dot{x}^{sys}(t) = \boldsymbol{A}^{sys}x^{sys} + \boldsymbol{F}(x^{air}, d) + \boldsymbol{B}^{sys}u + \Xi(t,p,d), \tag{3}$$

$$x^{sys} \in X, \ u \in U, \ u_{\min} \le u \le u_{\max}, \ p \in P, \ d \in D, \ A^{sys} \in \mathbb{R}^{c\times c}, \ B^{sys} \in \mathbb{R}^{c\times m}.$$

Some parameters are slow varying. However, the *fast* changes in the adverse rapidly-changing environments and flight conditions must be also considered. We consistently represent nonlinearities, parameter variations and uncertainties, thereby coherently solving design problems.

Lemma 1. A function $f(t, x)$ is bounded within a sector $[a,b]$ in $\mathbb{R}_+ \times \mathbb{R}^c$ – dimensional space, and, $f(t, x)$ evolves in $[a, b]$ if for $f(\cdot): \mathbb{R}_+ \times \mathbb{R}^c \to \mathbb{R}^c$, there exist real $a, b \in \mathbb{R}(a < b)$, such that

$$f(t,0) = 0, \ \forall t \in \mathbb{R}_+,$$

and, $[f(t,x) - ax]^T[bx - f(t,x)] \ge 0, \ \forall t \in \mathbb{R}_+, \forall x \in \mathbb{R}^c.$

The conditions of *Lemma 1* are always guaranteed for realistic aircraft nonlinearities in $x^{sys} \in X, \ u \in U, \ p \in P$ and $d \in D.$

State Transformation Method: Tracking Control. To ensure the specified performance and capabilities, closed-loop flight systems should be reconfigurable to guarantee real-time or near-real-time reconfigurability and adaptability [4; 5]. We design control laws, examine closed-loop system stability and study robustness under the bounded uncertainties $p \in P$ and perturbations $d \in D$.

Our goals are to: (i) Design practical and implementable tracking control laws; (ii) Develop a coherent design concept which will ensure a cohesive solution of nonlinear optimization problem using nonlinear equations of motion and minimizing nonquadratic functionals.

The objectiveis to develop a control procedure which may enable near-real-time reconfiguration and adaptation ensuring:

– Robustness to parameter variations;

– Disturbance attenuation;

– Stability despite failures or damages if the aircraft remains controllable and stabiliazable under these failures or damages in the considered flight envelope.

1. Tracking Control of Linear Systems. The tracking control problem is solved for linear systems by designing the proportional-integral control laws using the *state transformation* method [7].

The output and tracking error equations are given as

$$y(t) = Hx^{sys}(t),\ e(t) = Nr(t) - y(t) = Nr(t) - Hx^{sys}(t), \tag{4}$$

where $e \in E \subset \mathbb{R}^{b}$, $r \in R \subset \mathbb{R}^{b}$, $y \in Y \subset \mathbb{R}^{b}$, $N \in \mathbb{R}^{b\times b}$.

To enable the stable evolution of the tracking error, we define

$$\dot{e}(t) = -A_E I_E e - HA^{sys}x^{sys} - HB^{sys}u, \tag{5}$$

where $A_E \in \mathbb{R}^{b\times b}$ and $I_E \in \mathbb{R}^{b\times b}$ are the diagonal and identity matrices.

For linear systems $F(x^{air}, d) = 0$. From (3), we have

$$\dot{x}^{sys}(t) = Ax^{sys} + Bu + \Xi(t, p, d),\ y(t) = Hx^{sys}(t), \tag{6}$$

We use the results of *Lemma 1*. Using the expanded state vector $x = [x^{sys} e]^T$, if $\Xi(t, p, d) = 0$, one finds

$$\dot{x}(t) = \begin{bmatrix} \dot{x}^{sys}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} A^{sys} & 0 \\ -HA^{sys} & -A_E I_E \end{bmatrix}\begin{bmatrix} x^{sys} \\ e \end{bmatrix} + \begin{bmatrix} B^{sys} \\ -HB^{sys} \end{bmatrix}u + \begin{bmatrix} 0 \\ N \end{bmatrix}\dot{r} = Ax + Bu + \begin{bmatrix} 0 \\ N \end{bmatrix}\dot{r}. \tag{7}$$

Determine the evolution of the control function as

$$\dot{u} = -A_U I_U u + I_U v, \tag{8}$$

where $A_U \in \mathbb{R}^{m\times m}$ and $I_U \in \mathbb{R}^{m\times m}$ are the diagonal and identity matrices.

The *space transformation* method implies the use of the *state* vectors *z* and *v*. We define these vectors as

$$z = \begin{bmatrix} x & u \end{bmatrix}^T,\ v = u + \dot{u}. \tag{9}$$

Using *z* and *v*, one obtains the system model as

$$\dot{z}(t) = \begin{bmatrix} A & B \\ 0 & -A_U I_U \end{bmatrix} z + \begin{bmatrix} 0 \\ A_U I_U \end{bmatrix} v = A_z z + B_z v,\ y = Hx^{sys}. \tag{10}$$

We minimize the quadratic functional

$$J = \frac{1}{2}\int_{t_0}^{t_f}\left(z^T Q_z z + v^T G_z v\right)dt, \tag{11}$$

where $Q_z \in \mathbb{R}^{(c+m)\times(c+m)}$, $Q_z \geq 0$ and $G_z \in \mathbb{R}^{m\times m}$, $G > 0$.

Problem Formulation: Linear Systems. Minimize the quadratic functional (11) subject to (10), and, derive the control law for linear system (6).

We apply the Hamilton-Jacobi concept. From

$$H = \frac{1}{2}\left(z^T Q_z z + v^T G_z v\right) + \frac{\partial V^T}{\partial z}\left(A_z z + B_z v\right), \tag{12}$$

the first-order necessary condition for optimality $\frac{\partial H}{\partial z} = 0$ gives

$$v = -G_z^1 B_z^T \frac{\partial V}{\partial z}. \tag{13}$$

The solution of the Hamilton-Jacobi equation

$$-\frac{\partial V}{\partial t} = \frac{1}{2} z^T Q_z z + \left(\frac{\partial V}{\partial z}\right)^T A_z z - \frac{1}{2}\left(\frac{\partial V}{\partial z}\right)^T B_z G_z^{-1} B_z^T \frac{\partial V}{\partial z} \tag{14}$$

is satisfied by the continuous differentiable quadratic function

$$V(z) = \frac{1}{2} z^T K z,\ K \in \mathbb{R}^{(c+m)\times(c+m)}. \tag{15}$$

From (14) and (15), the equation for the positive-definite matrix K is found to be

$$-\dot{K} = KA_z + A_z^T K - KB_z G_z^{-1} B_z^T K + Q_z,\ K(t_f) = K_f. \tag{16}$$

From (13) and (15), recalling $z = \begin{bmatrix} x & u \end{bmatrix}^T$, the control function is

$$v = -G_z^{-1} B_z^T K z = -G_z^{-1} B_z^T K \begin{bmatrix} x \\ u \end{bmatrix}. \tag{17}$$

From (9), we have

$$\begin{aligned} \dot{u}(t) &= -G_z^{-1} B_z^T K z - I_U u = -G_z^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix}^T \begin{bmatrix} K_{11} & K_{21}^T \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} - A_U I_U u = \\ &= -G_z^{-1} K_{21} x - \left(G_z^{-1} K_{22} - A_U I_U\right) u = K_{f1} x + K_{f2} u. \end{aligned} \tag{18}$$

Using (7), one finds

$$u = B^{-1}\left(\dot{x}(t) - Ax\right) = \left(B^T B\right)^{-1} B^T \left(\dot{x}(t) - Ax\right). \tag{19}$$

Thus, we obtain

$$\begin{aligned} \dot{u}(t) &= K_{f1} x + K_{f2} u = K_{f1} x + K_{f2}\left(B^T B\right)^{-1} B^T \left(\dot{x}(t) - Ax\right) = \\ &= \left[K_{f1} - K_{f2}\left(B^T B\right)^{-1} B^T A\right] x(t) + K_{f2}\left(B^T B\right)^{-1} B^T \dot{x}(t) = \\ &= \left(K_{f1} - K_{F1} A\right) x(t) + K_{F1}\dot{x}(t) = K_{F2} x(t) + K_{F1}\dot{x}(t). \end{aligned} \tag{20}$$

From (20), a proportional-integral tracking control law with state feedback is found to be

$$u(t) = K_{F1} x(t) - K_{F1} x_0 + \int K_{F2} x(\tau) d\tau + u_0,\quad x = \begin{bmatrix} x^{sys} \\ e \end{bmatrix}. \tag{21}$$

2. Tracking Control of Nonlinear Systems

For nonlinear systems (1) and (3), applying the design procedure reported, one obtains the following control function

$$v = -G_z^{-1} B_z^T \frac{\partial V}{\partial z}, \quad V(z) = \sum_i v^{2i}, \quad i = 1, 2, ... \tag{22}$$

The derived (22) yields proportional-integral control law $u(t)$. The solution of the Hamilton-Jacobi functional differential equations must be found by using the nonquadratic positive-definite return function $V(z)$ [4; 8]. For example,

$$V(z) = \frac{1}{2} z^T K_1 z + \frac{1}{4}\left(z^T K_2 z\right)\left(z^T K_2 z\right) + ... \tag{23}$$

To avoid complexity, an alternative solution is derived.

For nonlinear systems (1) and (3), using the *state transformation* vectors z and v, one obtains

$$\dot{z}(t) = \begin{bmatrix} A & B \\ 0 & -A_U I_U \end{bmatrix} z + F(z) + \begin{bmatrix} 0 \\ A_U I_U \end{bmatrix} v = \tag{24}$$
$$= A_z z + F(z) + B_z v, \quad y = Hx^{sys}, \quad z(t_0) = z_0.$$

We use the following nonquadratic functional

$$J = \frac{1}{2} \int_{t_0}^{t_f} \left(z^T Q_z z - \frac{\partial V^T}{\partial z} F(z) + v^T G_z v \right) dt. \tag{25}$$

<u>*Problem Formulation: Nonlinear Systems.*</u>

Minimize the nonquadratic functional (25) subject to (24), and, derive the control law for the nonlinear system (3).

The proposed nonquadratic functional (25) yields the functional equation (14) which is satisfied by the quadratic Lyapunov function (15). The control law is given by (21).

3. Tracking Control of Nonlinear Uncertain Systems

The parameter uncertainties, perturbations and disturbances were used in the equations of motion (2) and (3). One may obtain $A_z(p, d)$ and $B_z(p, d)$ in (7) and (24). The norm of evolutions of the bounded uncertainties $p \in P$ and perturbations $d \in D$, given as $\Xi(t, p, d)$, is bounded. That is,

$$\|\Xi(t, p, d)\| \le \rho(t, x), \tag{26}$$

where $\rho(\cdot): \mathbb{R}_{\ge 0} \times \mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$ is the continuous Lebesgue measurable function. This fact was formulated by *Lemma 1*.

Under uncertainties and perturbation, we obtain

$$\dot{z}(t) = \begin{bmatrix} A & B \\ 0 & -A_U I_U \end{bmatrix} z + F(z) + \Xi_z + \begin{bmatrix} 0 \\ A_U I_U \end{bmatrix} v = A_z z + F(z) + \Xi_z + B_z v. \tag{27}$$

Consider the closed-loop system (22) – (27). The state variable and tracking error vectors evolve in $XE(X_0, E_0, U, R, Y, P, D) \subset \mathbb{R}^c \times \mathbb{R}^b$.

The functional (25) must be positive-definite. That is, $J > 0,\ \forall x \in X,\ \forall u \in U,\ \forall r \in R,$ $\forall p \in P,\ \forall d \in X,\ \forall t \in [t_0, \infty).$

The admissible domain of robust stability and tracking S_c

$$S_c(\delta) = \Big\{ e \in \mathbb{R}^b : e_0 \in E_0 x \in X\left(X_0, U, R, Y, P, D\right),\ r \in R,\ y \in Y,\ p \in P,\ d \in D,\ t \in [t_0, \infty) \Big|$$
$$\|e(t)\| \le \rho_e\left(t, \|e_0\|\right) + \rho_r\left(\|r\|\right) + \rho_y\left(\|y\|\right) + \delta,\ \delta \ge 0,\ \forall e \in E(E_0, R, Y, P, D),\ \forall t \in [t_0, \infty) \Big\} \subset \mathbb{R}^b$$

is found using the criteria imposed on the Lyapunov function

$$\rho_1\|x\| \le \rho_2\|e\| \le V(t,x,e) \le +\rho_3\|x\| + \rho_4\|e\|,$$
$$\frac{dV(t,x,e)}{dt} \le -\rho_5\|x\| - \rho_6\|e\|, \tag{28}$$

where $\rho_e(\cdot):\mathbb{R}_{\ge 0}\times\mathbb{R}_{\ge 0}\to\mathbb{R}_{\ge 0}$ is the *KL*-function; $\rho_r(\cdot):\mathbb{R}_{\ge 0}\to\mathbb{R}_{\ge 0}$, $\rho_d(\cdot):\mathbb{R}_{\ge 0}\to\mathbb{R}_{\ge 0}$ and $\rho_y(\cdot):\mathbb{R}_{\ge 0}\to\mathbb{R}_{\ge 0}$ are the *K*-functions [8].

Theorem. For nonlinear systems (3), the proportional-integral tracking control law (21) is derived by minimizing nonquadratic functional (25) subject the system dynamics. The robust tracking, stability and disturbance attenuation in *XE* are guaranteed if $XE \subseteq S_s$ for given initial conditions ($x_0 \in X_0$ and $e_0 \in E_0$), control bounds $u \in U$, references $r \in R$, uncertainties $p \in P$ and perturbations $d \in D$. All solutions of the closed-loop system $x(\cdot):[t_0,\infty)\to\mathbb{R}^c$ and evolutions $e(\cdot):[t_0,\infty)\to\mathbb{R}^b$ are robustly bounded if (28) are guaranteed. The convergence of the tracking error vector $e(\cdot):[t_0,\infty)\to\mathbb{R}^b$ to $S_e(\delta)$ is guaranteed if $XE \subseteq S_e$.

Design of Tracking Control Laws. Consider a twin-tail super-maneuverable multirole fighter aircraft. Using the governing equations of motion (3), nonlinearities and matrices of coefficients for an unbalanced and asymmetric aircraft at Mach number 0,5 and altitude 5000 m were derived in [4 – 10]. In particular

$$F\left(x^{sys}\right) = \begin{bmatrix} 0 \\ -p\cos\alpha\tan\beta - r\sin\alpha\tan\beta \\ \dfrac{1}{I_Y}\left[\left(I_Z - I_X\right)pr - I_{XZ}p^2 + I_{XZ}r^2\right] \\ q\cos\phi - r\sin\phi \\ p\sin\alpha - r\cos\alpha \\ \dfrac{1}{I_X I_Z - I_{XZ}^2}\left[I_{XZ}\left(I_X - I_Y + I_Z\right)qp + \left(I_Y I_Z - I_{XZ}^2 - I_Z^2\right)qr\right] \\ \dfrac{1}{I_X I_Z - I_{XZ}^2}\left[\left(I_X^2 - I_X I_Y + I_{XZ}^2\right)qp - I_{XZ}\left(I_X - I_Y + I_Z\right)qr\right] \\ q\tan\theta\sin\phi + r\tan\theta\cos\phi \\ q\cos^{-1}\theta\sin\phi + r\cos^{-1}\theta\cos\phi \end{bmatrix} \tag{29}$$

$$A^{sys} = \begin{bmatrix} -0,016 & 8,4 & -0,9 & -9,6 & -1,5 & -0,27 & -0,086 & 0 & -1 \\ -0,003 & -1,2 & 1 & 0 & 0,08 & 0,062 & 0,009 & -1 & 0 \\ -0,0001 & 3,9 & -0,85 & 0 & 0,017 & 0,0038 & 0,04 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0,003 & 0,15 & 0,02 & 0,001 & -0,56 & 0,13 & -0,91 & 0 & 0 \\ -0,00001 & 0,71 & 0,03 & 0,01 & -4,8 & -3,5 & 0,22 & 0 & 0 \\ 0,00001 & -0,94 & 0,06 & 0,005 & 9,2 & -0,028 & -0,51 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B^{sys}=\begin{bmatrix} 0,12 & 0,12 & -0,38 & -0,38 & 0 & 0 \\ -0,16 & -0,16 & -0,270 & -0,270 & 0,45 & 0 \\ -9,5 & -9,5 & -2,5 & -2,5 & 0,85 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0,019 & -0,019 & -0,001 & 0,001 & 0,42 & -0,053 \\ -2,9 & 2,9 & -3,1 & 3,1 & 0,73 & 0,92 \\ 3,1 & -3,1 & 0,78 & -0,78 & 0,61 & -0,45 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

I_X = 21983 kg·m^2, I_Y = 154248 kg·m^2, I_Z = 186515 kg·m^2 and I_{XZ} = 2407 kg·m^2. The mechanical limits on the deflections of control surfaces are accounted. In particular, $|\delta_{HR}, \delta_{HL}| \le 0{,}5$ rad, $|\delta_{FR}, \delta_{FL}| \le 0{,}4$ rad, $|\delta_C| \le 0{,}6$ rad and $|\delta_R| \le 0{,}5$ rad.

4. Tracking Control Law: Linearized Differential Equations. Tracking control laws are designed for the linearized model (7). We let $A_E = I$ and $A_U = I$. To demonstrate the concept, we first design control laws for decoupled longitudinal and lateral models.

4.1. Longitudinal Model

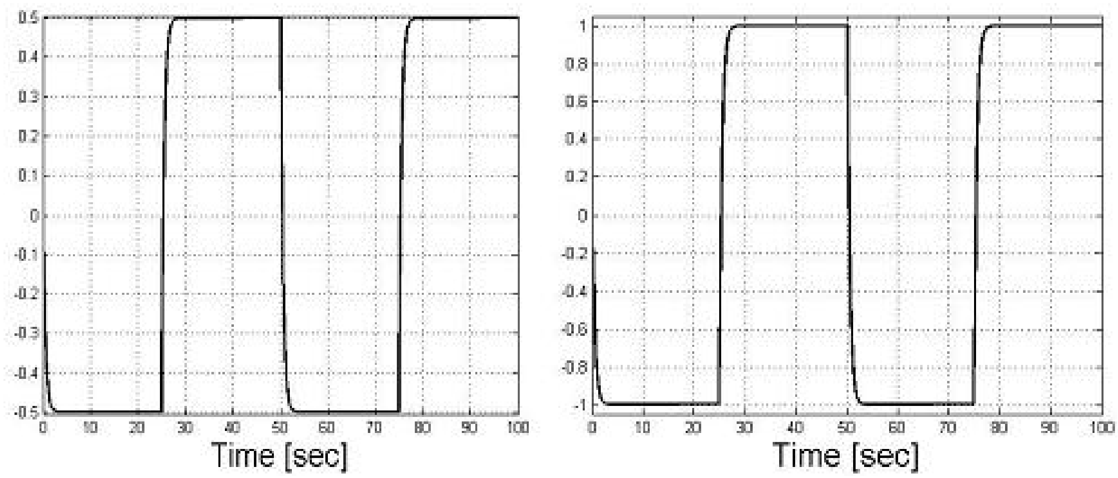
Decoupling gives the governing equation (3) with matrices

$$A^{sys}=\begin{bmatrix} -0,016 & 8,4 & -0,9 & -9,6 \\ -0,003 & -1,2 & 1 & 0 \\ -0,0001 & 3,9 & -0,85 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B^{sys}=\begin{bmatrix} 0,12 & 0,12 \\ -0,16 & -0,16 \\ -9,5 & -9,5 \\ 0 & 0 \end{bmatrix}.$$

The control law (21) is derived using the weighting coefficients $q_{z1,1} = q_{z2,2} = q_{z3,3} = q_{z4,4} = q_{z5,5} = 1\times10^{11}$, $q_{z6,6} = q_{z7,7} = 1\times10^{6}$ and $G_z = 1\times10^{4} I$, $I \in \mathbb{R}^{2\times2}$. In (21), the matrices K_{F1} and K_{F2} are

$$K_{F1}=\begin{bmatrix} -0,046 & 0,061 & 3,63 & 0 & 0 \\ -0,046 & 0,061 & 3,63 & 0 & 0 \end{bmatrix}, \quad K_{F2}=\begin{bmatrix} -0,0013 & 0,204 & 127 & 1,83 & -2109 \\ -0,0013 & 0,204 & 127 & 1,83 & -2109 \end{bmatrix}.$$

For a closed-loop system, the evolutions of the pitch angle θ for different θ_{ref} are reported in fig. 1.



a *b*

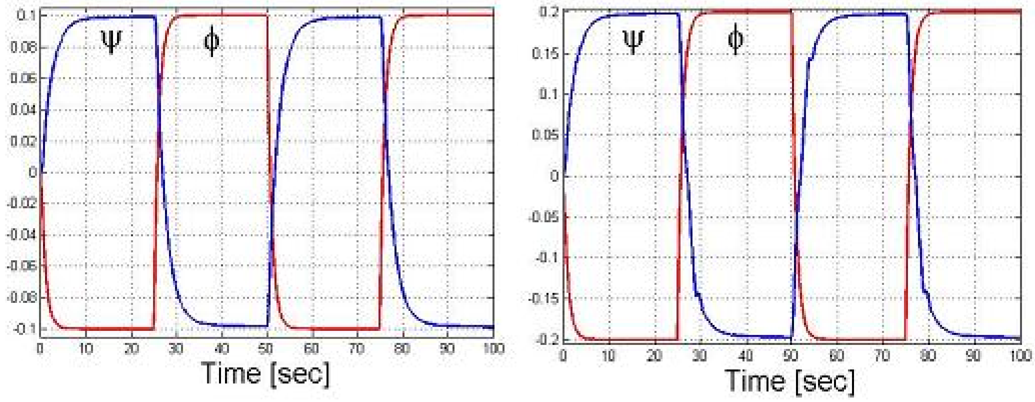
Fig. 1. Evolution of the output θ: (*a*) – θ_{ref} = 0,5 rad; (*b*) – θ_{ref} = 1 rad

4.2. Lateral Model

Matrices A^{sys} and B^{sys} are found from (29). The control law (21) is derived assigning $q_{z1,1} = q_{z2,2} = q_{z3,3} = q_{z4,4} = q_{z5,5} = 1$, $q_{z6,6} = 1\times10^6$, $q_{z7,7} = 1\times10^8$, $q_{z8,8} = q_{z9,9} = q_{z10,10} = 1\times10^6$ and $G_z = I$, $I \in \mathbb{R}^{3\times3}$. We found the following matrices

$$K_{F1} = \begin{bmatrix} -173 & 3218 & 6366 & 0 & 0 & 0 & 0 \\ 173 & -3218 & -6366 & 0 & 0 & 0 & 0 \\ -1105 & 1085 & 43104 & 0 & 0 & 0 & 0 \end{bmatrix}, \; K_{F2} = \begin{bmatrix} 7752 & 11473 & -1088 & -0,21 & -0,68 & -4,6 & 37266 \\ -7752 & -11473 & 1088 & 0,21 & 0,68 & 4,6 & -37266 \\ 12010 & 40835 & 15867 & 0,96 & -0,29 & 109 & -1604 \end{bmatrix}.$$

Figures 2 document the roll and yaw angles ϕ and ψ for the closed-loop system for different references ϕ_{ref} and ψ_{ref}.

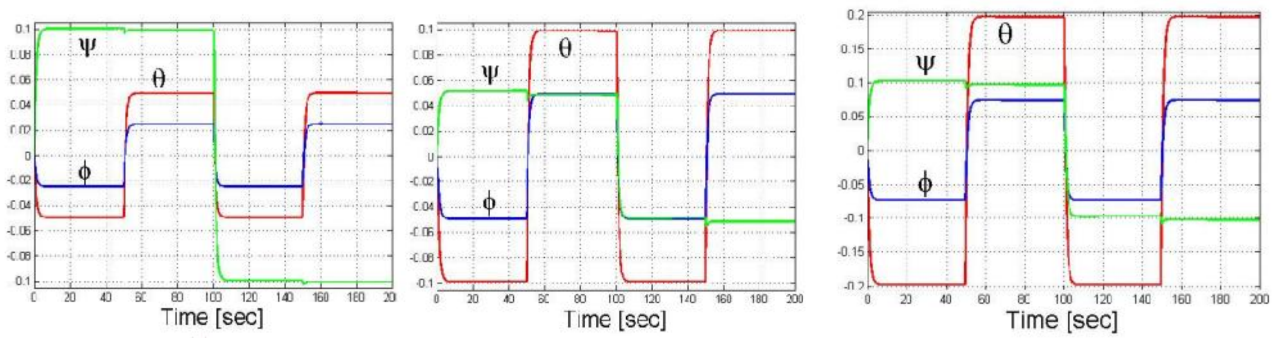


a *b*

Fig. 2. Evolutions of the outputs ϕ and ψ: (*a*) ϕ_{ref} = 0,1 and ψ_{ref} = – 0,1 rad; (*b*) ϕ_{ref} = 0,2 and ψ_{ref} = – 0,2 rad

4.3. Augmented Longitudinal and Lateral Dynamics

The linear equations of motion are given as (7). The weighting coefficients of diagonal matrices Q_z and G_z are assigned using $q_{zii} = 1/z^2_{i\max}$ and $g_{zii} = 1/v^2_{i\max}$. Let $q_{z1,1} = q_{z2,2} = q_{z3,3} = q_{z4,4} = q_{z5,5} = q_{z6,6} = q_{z7,7} = q_{z8,8} = q_{z9,9} = 1$, $q_{z10,10} = q_{z11,11} = 1\times10^{11}$, $q_{z12,11} = 1\times10^{10}$, $q_{z13,13} = q_{z14,14} = q_{z17,17} = 1\times10^6$, $q_{z15,15} = q_{z16,16} = q_{z18,18} = 1\times10^7$ and $G_z = 100I$, $I \in \mathbb{R}^{6\times6}$, $g_{zii} = 100$. The control law (21) is designed. Figures 3 report evolutions of the outputs if $r = [\theta_{ref}, \phi_{ref}, \psi_{ref}]^T$. The simulation results illustrate that the desired Euler angles are achieved.

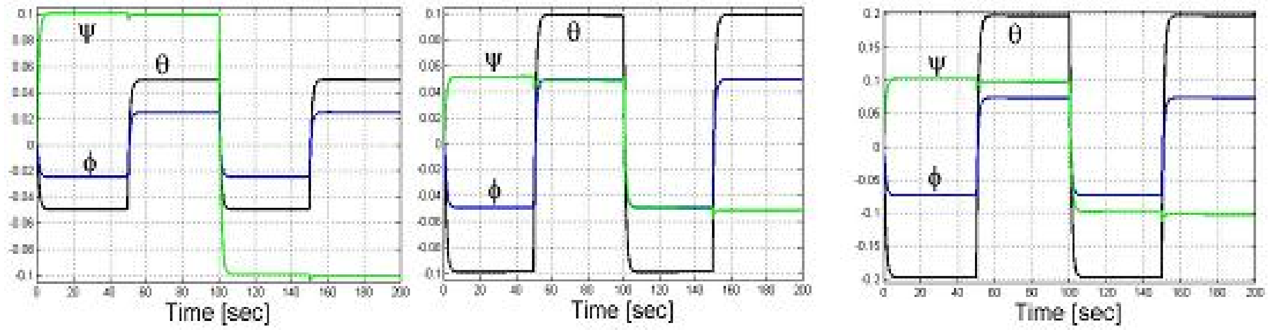


a *b* *c*

Fig. 3. Evolutions of outputs θ, ϕ and ψ for $r = [\theta_{ref}, \phi_{ref}, \psi_{ref}]^T$: (*a*) – $r = [0,05 \;\; 0,025 - 0,1]^T$ rad; (*b*) – $r = [0,1 \;\; 0,05 - 0,05]^T$ rad; (*c*) – $r = [0,2 \;\; 0,075 - 0,1]^T$ rad

4.4. Nonlinear Fighter Model

The tracking control law (21) is designed. The matrix K and feedback coefficients are found. Figures 4 report the evolutions of the outputs for a nonlinear fighter model with saturations on deflections of control surfaces. Good dynamic performance and tracking is achieved for different references.



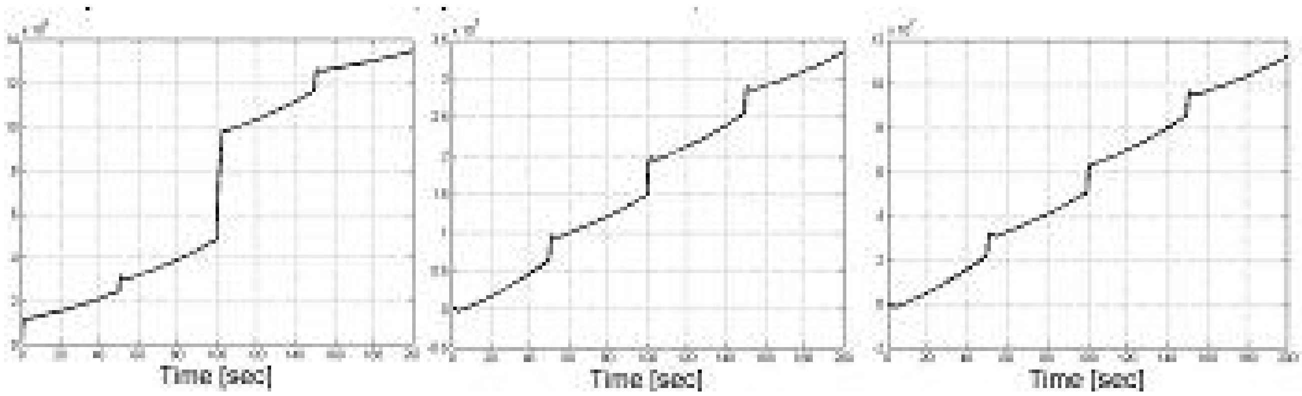
a *b* *c*

Fig. 4. Evolutions of outputs θ, ϕ and ψ for $r = [\theta_{ref}, \ \phi_{ref}, \ \psi_{ref}]^T$: (*a*) – $r = [0{,}05 \ \ 0{,}025 - 0{,}1]^T$rad; (*b*) – $r = [0{,}1 \ \ 0{,}05 - 0{,}05]^T$ rad; (*c*) – $r = [0{,}2 \ \ 0{,}075 - 0{,}1]^T$ rad

The robustness to uncertainties and parameter variations (including variations of the inertia moment) is examined in the operating envelope *XE*. The robustness and stability are verified using the admissibility concept by applying the necessary and sufficient conditions as formulated by Theorem. The conditions (28) imposed on Lyapunov function are met. The positive-definiteness of performance functional (25) is examined for admissible $r \in R$ in the operating envelope with $p \in P$ and $d \in D$. The closed-loop system ensure robustness and stability. We study the evolutions and positive definiteness of

$$J = \frac{1}{2}\int_{t_0}^{t_f}\left(z^T Q_z z - z^T K F(z) + v^T G_z v\right)dt. \tag{30}$$

The fighter performance and capabilities are evaluated in a realistic operating envelope *XE*. Figures 5 illustrate the evolution of $\frac{1}{2}\int\left(z^T Q_z z + v^T G_z v\right)dt$, $\int\left(z^T K F(z)\right)dt$ and a total functional J (30). It is evident that $J > 0$ in the evolution set *XE* (X_0, E_0, *U*, *R*, *Y*, *P*, *D*). The *XE* depends on the initial conditions, references, disturbances, etc. Robustness and stability of a closed-loop system may be guaranteed in the expanded *XE* despite of uncertainties, perturbations, etc.



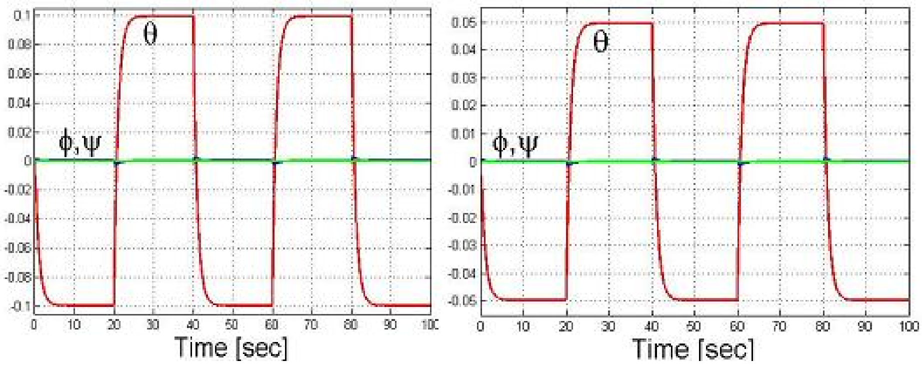
a *b* *c*

Fig. 5. Evolutions of the performance functional *J*: (*a*) – $r = [0{,}05 \ \ 0{,}025 - 0{,}1]^T$ rad; (*b*) $r = [0{,}1 \ \ 0{,}05 - 0{,}05]^T$rad; (*c*) $r = [0{,}2 \ \ 0{,}07 - 0{,}1]^T$rad

5. Accommodation of Failures. We examine the abilities of a closed-loop system to accommodate some failures and damages of control surfaces and airframe. Our results demonstrate that the aircraft may be stabilized and controlled: (i) In some operating envelopes XE; (ii) Under some failures. In general, aircraft may become unstable and uncontrollable. Due to distinct failures, damages and flight scenarios, it is very difficult to coherently define operating envelopes XE for which the condition $XE \subseteq S_e$ is guaranteed. The evolutions of states and outputs can be unbounded. Under some $p \in P$ and $d \in D$, which correspond to particular damages and flight envelopes, it is impossible to find stabilizing control laws which may not exist. One may reconfigure control law (21) in near-real-time.

For the failed flaps scenario, assume that the damaged flaps are positioned as $\delta_{FR} = \delta_{FL} = 0$. The system nonlinearities and control constraints are considered. The control law (21) is redesigned. The evolution of the outputs are documented in fig. 6 if $r = [\theta_{ref}, 0, 0]^T$.

We perform numerical studies and simulations to substantiate the design concept in XE for other failures. It is found that near-real-time reconfiguration can be achieved because the identification takes ~0,0073 sec, while, the control law redesign requires ~0,02 sec. Optimizing numerical algorithms, using the initial values, utilizing advanced processing platforms, near-real-time adaptation can be ensured.



a *b*

Figure 6. Evolutions of θ, ϕ and ψ for flaps failure, r = $[\theta_{ref}, \phi_{ref}, \psi_{ref}]^T$:
(a) – Linear system, $r = [0{,}1\ 0\ 0]^T$rad; (b) – Nonlinear system, $r = [0{,}05\ 0\ 0]^T$rad

Conclusions. The overall objective was to develop, demonstrate and substantiate robust, practical, computationally effective and numerically efficient design methods for closed-loop systems. We designed proportional-integral tracking control laws for flight vehicles by using Hamilton-Jacobi concept and Lyapunov stability theory. The *space transformation* method was applied. Near-real-time control law redesign, performed periodically within allowable time, and controller reconfiguration may ensure adaptation, controllability and stability even under some failures and damages. Enabled maneuverability, increased agility, enhanced controllability, stability and robustness, as well as improved flying and handling qualities were achieved by using the proposed design scheme and derived control algorithms. The results were verified through nonlinear simulations in expanded operating envelopes. Our findings are applicable to various flight and aerospace systems including a new generation of fighter aircraft, unmanned aerial vehicles, etc. This paper developed, applied and verified new control schemes and methods providing new inroads and solutions for flight control system design and optimization.

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Авніш П. С. Чохан, С. Е. Лішевскій

Аналіз нелінійної динаміки сучасних літаків на основі методу перетворення стану

Розглянуто питання нелінійної динаміки, оптимізації та керування поздовжніми і бічними рухами літальних апаратів. Для вдосконалених планерів при наявності багатьох керуючих поверхонь неможливо лінеаризувати і розділити поздовжню і бічну динаміку. Виконано нелінійний аналіз системи керування. Для прийнятого планера і схем керування досліджено концепцію робастного проектування. Узгоджене проектування застосовується з мінімальними спрощеннями і припущеннями. Перевірено висновки і продемонстровано під час вирішення практичних завдань. Запропонований метод проектування можна здійснити майже в реальному часі, завдяки концептуальній послідовності, робастності, обчислювальній та алгоритмічній ефективності. Запропонована концепція ефективна у разі проектування систем автоматичного літаководіння та керування польотом.

Авниш П. С. Чохан, С. Э. Лишевский

Анализ нелинейной динамики современных самолетов на основе метода преобразования состояния

Рассмотрены вопросы нелинейной динамики, оптимизации и управления продольными и боковыми движениями летательных аппаратов. Для усовершенствованных планеров при наличии многих управляющих поверхностей невозможно линеаризовать и разделить продольную и боковую динамику. Выполнен нелинейный анализ системы управления. Для принятого планера и схем управления исследована концепция робастного проектирования. Согласованное проектирование применяется с минимальными упрощениями и допущениями. Проверены выводы и продемонстрированы при решении практических задач. Предложенный метод проектирования может быть осуществлен почти в реальном времени, благодаря концептуальной последовательности, робастности, вычислительной и алгоритмической эффективности. Предложенная концепция эффективна при проектировании систем автоматического самолетовождения и управления полетом.