UDC 681.51(045)

Younis Shareef Daoud, Dr.

NEW APPROARCH FOR THE SOLUTION OF QUADROTOR STABILIZATION TASK

The requirements for the functional possibilities of unmanned air vehicles are determined. It is marked the necessity of their control autonomy. It is offered to increase the place of unmanned air vehicles autonomy system of control, shortening here the loads that the pilot on the ground is tested. The approach for the statement task solution based on the use of mathematical model of quadrotor and quality quadratic criterion is considered.

Keywords: uninhabited air vehicle, quad-rotor system, navigation and control algorithm.

Introduction. In recent years, there are an increasing amount of researches on automatic flying of intelligent systems. Those systems are generally called as flying robots or unmanned air vehicle (UAV). The uninhabited air vehicles are defined as aircraft without the onboard presence of pilots [1]. Today, lots of different UAVs model are available, and those structures are named with respect to rotor number or physical appearance. Those systems are widely used for military applications, search and rescue operations, agricultural disinfection, filming sports events or movies from almost any angle and transporting or controlling equipment.

In this project; a VTOL (Vertical Take Off and Landing) flying robot which has a Quadrotor system type is desired to design.

Statement of the task. The structure scheme of a quadrotor system dynamic model is represented on fig. 1.

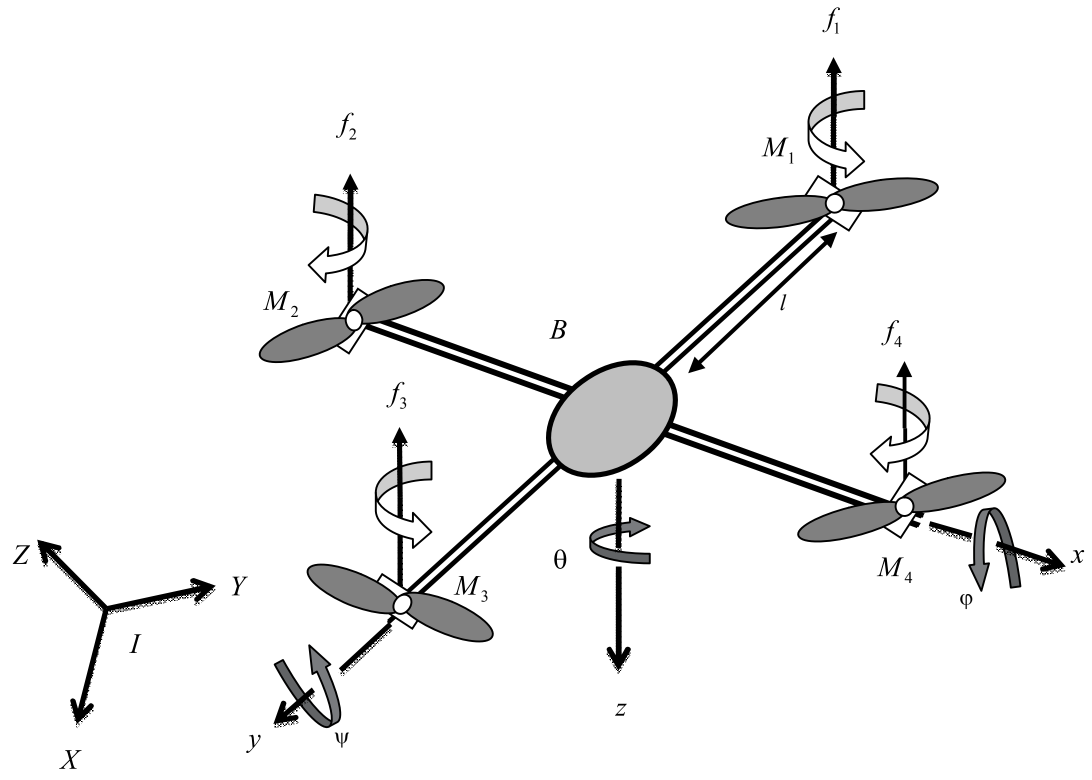


Fig. 1. Structure scheme of a quadrotor system dynamic model

The dynamics of the four rotors are relatively much faster than the main system and thus neglected in our case. The generalized coordinates of the rotorcraft are $q = (x, y, z, \psi, \theta, \varphi)$, where

Dynamic model of the quadrotor in terms of position (x, y, z) and rotation (φ, θ, ψ) is written as [2]:

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -g \end{pmatrix} + \frac{1}{m} \begin{pmatrix} -\sin\theta \\ \cos\theta\sin\varphi \\ \cos\theta\cos\varphi \end{pmatrix} u \qquad (1)$$

$$\begin{pmatrix} \ddot{\varphi} \\ \ddot{\theta} \\ \ddot{\psi} \end{pmatrix} = f(\varphi,\theta,\psi) + g(\varphi,\theta,\psi)\tau \qquad (2)$$

where, $f(\varphi,\theta,\psi) = \begin{pmatrix} \dot{\theta}\dot{\psi}\left(\dfrac{I_y - I_z}{I_x}\right) - \dfrac{J_p}{I_x}\dot{\theta}\Omega \\ \dot{\varphi}\dot{\psi}\left(\dfrac{I_z - I_x}{I_y}\right) - \dfrac{J_p}{I_y}\dot{\varphi}\Omega \\ \dot{\varphi}\dot{\theta}\left(\dfrac{I_x - I_y}{I_z}\right) \end{pmatrix}$, $g(\varphi,\theta,\psi) = \begin{pmatrix} \dfrac{l}{I_x} & 0 & 0 \\ 0 & \dfrac{l}{I_y} & 0 \\ 0 & 0 & \dfrac{l}{I_z} \end{pmatrix}$, $u \in R^1$

and $\tau = \begin{pmatrix} \tau_\varphi \\ \tau_\theta \\ \tau_\psi \end{pmatrix} \in R^3$ are the control inputs, (x, y, z) represents the relative position of the center of mass of the quadrotor with respect to an inertial frame $\Im$, and (ψ, θ, φ) are the three *Euler* angles representing the orientation of the rotorcraft, namely yaw-pitch-roll of the vehicle. $I_{x,y,z}$ are body inertia, J_p is propeller/rotor inertia and $\Omega = \omega_2 + \omega_4 - \omega_1 - \omega_3$, I is the body inertia matrix, g is the acceleration due to gravity, $u = f_1 + f_2 + f_3 + f_4$ and f_i' -s are described as $f_i = k_i\omega_i^2$, where k_i are positive constants and ω_i are the angular speed of the motor i.

Thus, the system is the form of an under actuated system with six outputs and four inputs.

It's applied dynamic inversion to the system given by (1) and (2) to achieve station-keeping tracking control for the position outputs (x, y, z, ψ).

It's selected the convenient output vector $y_1 = (z, \varphi, \theta, \psi)$ which makes the dynamic inverse easy to find. Dynamic inversion now yields effectively an inner control loop that feedback linearizes the system from the control $u = (u, \tau_\varphi, \tau_\theta, \tau_\psi)$ to the output $x = (z, \varphi, \theta, \psi)$.

After linearization the equations (1), (2) can be represented as

$$\dot{x} = Ax + Bu \qquad (3)$$

with $x(t) \in R^n$ the state, $u(t) \in R^m$ the control input.

The equation of measurement is given by

$$y = Cx,$$

where $y(t) \in R^p$ the measured output.

The controls will be output feedbacks of the form

$$u = -Ky, \qquad (4)$$

where K is an $m \times p$ matrix of constant feedback coefficients to be determined by the design procedure.

The objective of state regulation for the quad-rotor is to drive any initial condition error to zero, thus guaranteeing stability. This may be achieved by selecting the control input $u(t)$ to minimize a quadratic *cost* or *performance index* (*PI*) of the type

$$J = \frac{1}{2}\int_0^\infty \left(x^{\mathrm{T}}Qx + u^{\mathrm{T}}Ru\right)dt, \tag{5}$$

where Q and R are symmetric positive semidefinite *weighting matrices*. Positive semidefiniteness of a square matrix M (denoted $M \geq 0$) is equivalent to all its eigenvalues being nonnegative, and also to the requirement that the quadratic form $x^{\mathrm{T}}Mx$ be nonnegative for all vectors x. Therefore, the definiteness assumptions on Q and R guarantee that J is nonnegative and lead to a sensible minimization problem. This quadratic *PI* is a vector version of an integral-squared *PI* of the sort used in classical control [3].

Formulate the linear quadratic regulation (LQR) problem with output feedback as find the feedback coefficient matrix K in the control input (4) that minimizes the value of the quadratic *PI* (5) [4].

By substituting the control (4) into (3) the closed-loop system equations are found to be

$$\dot{x} = (A - BKC)x \equiv A_c x. \tag{6}$$

The *PI* may be expressed in terms of K as

$$J = \frac{1}{2}\int_0^\infty x^{\mathrm{T}}\left(Q + C^{\mathrm{T}}K^{\mathrm{T}}RKC\right)xdt. \tag{7}$$

The design problem is now to select the gain K so that J isminimized subject to the dynamical constraint (6).

This *dynamical* optimization problem may be converted into an equivalent *static* one that is easier to solve as follows. Suppose that wecan find a constant, symmetric, positive-semidefinite matrix P so that

$$\frac{d}{dt}(x^{\mathrm{T}}Px) = -x^{\mathrm{T}}\left(Q + C^{\mathrm{T}}K^{\mathrm{T}}RKC\right)x. \tag{8}$$

Then J may be written as

$$J = \frac{1}{2}x^{\mathrm{T}}(0)Px(0) - \frac{1}{2}\lim_{t\to\infty} x^{\mathrm{T}}(t)Px(t).$$

Assuming that the closed-loop system is asymptotically stable so that $x(t)$ vanishes with time, this becomes

$$J = \frac{1}{2}x^{\mathrm{T}}(0)Px(0). \tag{9}$$

If P satisfies (8), we may use (6) to see that

$$-x^{\mathrm{T}}\left(Q + C^{\mathrm{T}}K^{\mathrm{T}}RKC\right)x = \frac{d}{dt}(x^{\mathrm{T}}Px) = \dot{x}^{\mathrm{T}}Px + x^{\mathrm{T}}P\dot{x} = x^{\mathrm{T}}\left(A_c^{\mathrm{T}}P + PA_c\right)x.$$

Since this must hold for all initial conditions, and hence for all state trajectories $x(t)$, we may write

$$g \equiv A_c^{\mathrm{T}}P + PA_c + C^{\mathrm{T}}K^{\mathrm{T}}RKC + Q = 0. \tag{10}$$

If K and Q are given and P is to be solved for, this is called a *Lyapunov equation*.

It is now necessary to use this result to compute the gain K that minimizes the *PI*. By using the trace identity

$$\mathrm{tr(AB) = tr(BA)}$$

for any compatibly dimensioned matrices A and B (with the trace of a matrix the sum of its diagonal elements), we may write (9) as

$$J = \frac{1}{2}\mathrm{tr}(PX) \tag{11}$$

where the $n \times n$ symmetric matrix X is defined by

$$X \equiv \frac{1}{2} x(0) x^{\mathrm{T}}(0).$$

It is now clear that the problem of selecting K to minimize (7) subject to the dynamical constraint (6) on the states is equivalent to the *algebraic* problem of selecting K to minimize (11) subject to the constraint (10) on the auxiliary matrix P.

To solve this modified problem, we use the Lagrange multiplier approach [2] to modify the problem yet again. Thus adjoin the constraint to the PI by defining the Hamiltonian

$$\aleph = \mathrm{tr}(PX) + \mathrm{tr}(gS) \tag{12}$$

with S a symmetric $n \times n$ matrix of Lagrange multipliers which still needs to be determined. Then our constrained optimization problem is equivalent to the simpler problem of minimizing (12) without constraints. To accomplish this we need only set the partial derivatives of $\aleph$ with respect to all the independent variables *P, S,* and K equal to zero. Using the facts that for any compatibly dimensioned matrices *A, B,* and C and any scalar *y,*

$$\frac{\partial}{\partial B}\mathrm{tr}(ABC) = A^{\mathrm{T}} C^{\mathrm{T}}$$

and

$$\frac{\partial y}{\partial B^{T}} = \left[\frac{\partial y}{\partial B}\right]^{\mathrm{T}},$$

the necessary conditions for the solution of the LQR problem with output feedback are given by

$$0 = \frac{\partial \aleph}{\partial S} = g = A_c^{\mathrm{T}} P + P A_c + C^{\mathrm{T}} K^{\mathrm{T}} R K C + Q;$$

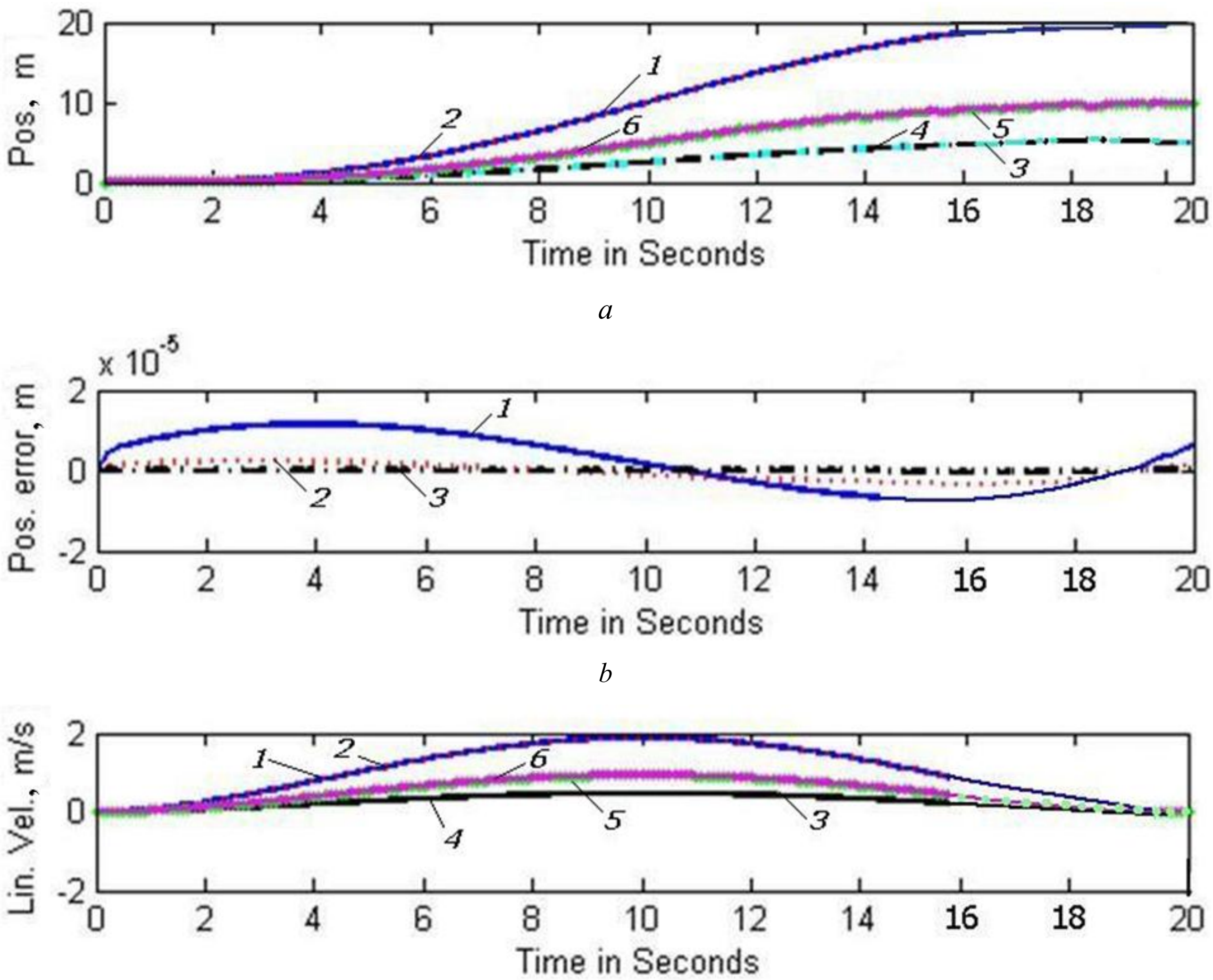
$$0 = \frac{\partial \aleph}{\partial S} = A_c S + S A_c^{\mathrm{T}} + X;$$

$$0 = \frac{1}{2}\frac{\partial \aleph}{\partial K} = RKCSC^{\mathrm{T}} - B^{\mathrm{T}} PSC^{\mathrm{T}}. \tag{13}$$

The first two of these are Lyapunov equations and the third is an equation for the gain K. If R is positive definite (i. e., all eigenvalues greater than zero, which implies nonsingularity; denoted $R > 0$) and CSC^{T} is nonsingular, then (13) may be solved for K to obtain

$$K = R^{-1} B^{\mathrm{T}} PSC^{\mathrm{T}} (CSC^{\mathrm{T}})^{-1}.$$

The results of simulations are presented on figure.



a

b

c

Fig. 2. Results of simulations: *a* – (*1* – x_3; *2* – x; *3* – y_3; *4* – x; *5* – z_3; *6* – z);
b – (*1* – $x_3 - x$; *2* – $y_3 - y$; *3* – $z_3 - z$); *c* – (*1* – $\dot{x}_3$; *2* – $\dot{x}$; *3* – $\dot{y}_3$; *4* – $\dot{y}$; *5* – $\dot{z}_3$; *6* – $\dot{z}$)

Conclusion. Mathematical model of the system will be obtained, some simulations will be applied and characteristics of the system will be observed.

References

1. *Das A.* Dynamic Inversion with Zero-Dynamics Stabilization for Quadrotor Control / A. Das, S. Subbarao, F. Lewis // IET Control Theory & Applications. – 2009. – Vol. 3, Iss. 3. – P. 303–314.
2. *Das A.* Backstepping Approach for Controlling a Quadrotor using Neural Network / A. Das, F. Lewis, S. Subbarao // Special Issue of Journal of Intellegent and Robotics Systems. – Springer, April 2009. – Vol. 56, Iss. 1. – 127 p.
3. Das A. Sliding Mode Approach to Control Quadrotor Using Dynamic Inversion, to appear as book chapter in Robust Control / A. Das, F. Lewis, Kamesh Subbarao // INTECH, 2011.
4. *P. Castillo* Stabilization of a mini rotorcraft having four rotors / P. Castillo, R. Lozano, A. Dzul // IEEE Control Systems Magazine. – Vol. 25, No. 6. – P. 45–55, regular paper, December, 2005.

Джони Шариф Дауд

Новый подход к решению задачи стабилизации QUAD-ротора

Определены требования к функциональным возможностям беспилотных летательных аппаратов, среди которых отмечена необходимость автономности их управления. Предлагается повысить роль автономной системы управления беспилотных летательных аппаратов, сократив при этом загрузки, которые испытывает пилот, находящийся на земле. Рассмотрен подход к решению поставленной задачи на основе использования математической модели QUAD-ротора и квадратичного критерия качества.

Джоні Шаріф Дауд

Новий підхід до вирішення завдання стабілізації QUAD-ротора

Визначено вимоги до функціональних можливостей безпілотних літальних апаратів, серед яких відмічено необхідність автономності їх керування. Пропонується підвищити роль автономної системи керування безпілотних літальних апаратів, скоротивши при цьому навантаження, які випробовує пілот, що перебуває на землі. Розглянуто підхід до вирішення поставленого завдання на основі використання математичної моделі QUAD-ротора і квадратичного критерію якості.