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ABOUT ALGORITHMS OF TARGET POSITIONING

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Abstract—*The article deals with the problem of determining the coordinates of the location of the target based on using some lines-of-sight by means of antennas mounted on unmanned aerial vehicles. Algorithms for target positioning using an unmanned aerial vehicle are proposed. The proposed algorithms require knowledge of the coordinates of several positions of the unmanned aerial vehicle. For solving the problem, the least-squares technique is used. Some ways of solving the problem are considered. In the first case, the algorithms assume finding several lines of sight of the target. Further, either a geometric or an analytical method for solving the problem is used. In the second case, the algorithm assumes measuring the distances from the vehicle to the target. The tasks are solved in vector and vector-matrix form. The results of simulation are presented, confirming the effectiveness of the proposed algorithms. The obtained results can be useful for both civilian and special areas of application.*

Index Terms—Algorithm; analytical and geometrical solutions; line-of-sight; positioning target; unmanned aerial vehicles.

I. INTRODUCTION

The scope of modern unmanned aerial vehicles (UAVs) affects both the civilian and special spheres: a reconnaissance of the terrain, environmental monitoring, security of protected objects, patrolling borders, traffic control, emergency assistance, etc [1]. Almost all of the listed UAV application areas require the use of onboard location systems [2], [3]. Such equipment allows determining the direction to a target, providing the necessary visual information about the observed objects. Therefore algorithms of target positioning are of great importance in topicality of unmanned aviation. Possibilities of modern UAVs allow solving some important problems. Determining the location of a target belongs to the above-mentioned ones. It should be noted that this problem has become especially important in the light of advances in the development of direction finders [4].

II. PROBLEM STATEMENT

Two ways of solving the problem are considered. In the first case, the algorithms assume finding several lines of sight of the target. Further, either a geometric or an analytical method for solving the problem is used.

The method of determination of the moving object's location depends on the type of position line to be used. Such an approach belongs to direction-finding techniques. The solution can be obtained in both graphical and analytical ways. The most widespread methods of determination of the moving object location are the direction-finding, the ranging navigation, the difference-ranging navigation, and the direction-ranging navigation [5].

The article deals with the problem of determining the coordinates of the target location based on using some lines-of-sight by means of antennas mounted on UAVs. The lines-of-sight are obtained by means of a camera mounted at the UAV. To solve this problem, it is necessary to know the coordinates of some UAV locations.

In the second case, the algorithm assumes measuring the distances from the aircraft to the target.

Theoretical fundamentals of calculating procedures for optimal synthesis of robust aircraft control systems based on the mixed H_2 / H_∞ approach are given in [3], [4]. This approach takes into consideration requirement to both robust performance and robustness of the synthesized system at the same time.

The algorithm of the synthesis of the systems assigned for stabilization of measuring-observation equipment operated on ground vehicles [5]. The full mathematical description of the stabilization system of ground vehicle equipment is represented in [6]. Analysis of the basic linearities of this mathematical description and approaches to linearization of the model are given in [7].

Features and results of simulating stabilization systems of ground vehicle measuring-observation equipment with continuous control unit are represented in [8]. Software for simulation of stabilization systems of ground measuring-observation equipment were researched in [9].

Ways of discretization of continuous control systems are described in many publications, for example, [10]. The possibilities of modern computer technique for discretization of continuous systems of the wide class are researched in [11]. Development of the stabilization system with a discrete control unit requires researching new approaches and specific research.

The goal of this article is determination of approaches to simulation of the stabilization system of ground vehicle equipment with a digital control unit and representation of simulation results based on these approaches.

III. PROBLEM SOLUTION

The problem of parametric optimization of systems of motion control of the wide class in general and stabilization system in particular requires using quality criteria in three aspects. Firstly, solving this problem requires forming the objective function. Secondly, optimization problems of the studied type require using a penalty function. Thirdly, a feature of these problems is the necessity of analysis of the obtained results with using various quality criteria.

1) Using Two Lines-of-Sight

In classical techniques, the position surface for a constant bearing of the moving point and for a constant bearing of the fixed point is the vertical plane passing through points *M* and *A*. The position line, that is the line, which passes through these points, is formed as a result of the intersection of this plane by the Earth's surface (the Earth's surface can be considered as a plane for small distances) [4].

Consider a problem of determining target coordinates (point *M* in Fig. 1) using two lines-of-sight obtained in two known locations of a UAV (points *A* and *B* in Fig. 1). Unit vectors of lines-of-sight are denoted $\vec{u}_A$ and $\vec{u}_B$ respectively. In practical situations, these vectors, as a rule, are given by two Euler angles [6].

The location of points *A* and *B* with known coordinates can be given by radius vectors $\vec{r}_A$ and $\vec{r}_B$. The location of the point *M* with unknown coordinates can be given by the radius vector $\vec{\rho}$.

The location of the point *M* relative to points *A* and *B* can be given by the radius vectors $\vec{d}_A$ and $\vec{d}_B$ respectively [8]. For problem statement, it is necessary to take into consideration the vector $\vec{s} = \vec{r}_B - \vec{r}_A$.

Consider two ways of solving the stated problem.

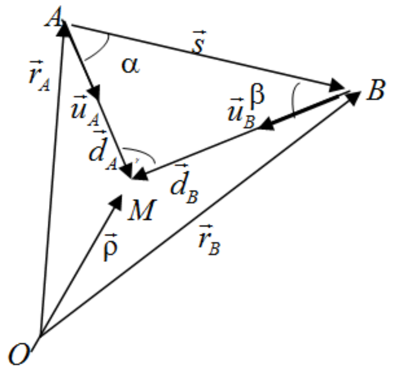

Fig. 1. Geometrical representation of a problem of determining target coordinates

2) Geometrical Solution

Based on the scheme represented in Fig. 1, we can write

$$\vec{\rho} = \vec{r}_A + \vec{d}_A = \vec{r}_A + d_A \vec{u}_A. \tag{1}$$

The problem reduces to determining the line segment d_A. In other words, it is necessary to obtain the side *AM* of the triangle *AMB*. It is necessary also to find angles α, β, and γ of this triangle. Based on the scheme shown in Fig. 1, it is possible to write

$$\cos\alpha = \vec{u}_A \cdot \frac{\vec{s}}{AB}. \tag{2}$$

In accordance with (2), an angle α can be determined as

$$\alpha = \arccos\left(\vec{u}_A \cdot \frac{\vec{s}}{AB}\right). \tag{3}$$

In a similar to (2), (3),

$$\beta = \arccos\left(\vec{u}_B \cdot \frac{-\vec{s}}{AB}\right). \tag{4}$$

Then

$$\gamma = 180^\circ - \alpha - \beta. \tag{5}$$

In accordance with the law of sines, we will obtain (see Fig. 1)

$$\frac{AB}{\sin\gamma} = \frac{d_A}{\sin\beta}, \tag{6}$$

Based on (6) and taking into consideration (5), we can write

$$d_A = AB\frac{\sin\beta}{\sin(\alpha+\beta)}. \quad (7)$$

3) Analytical Solution: Case 1

Based on the scheme given in Fig. 1, we can represent the radius vector $\vec{\rho}$ in two ways

$$\vec{\rho} = \vec{r}_A + d_A\vec{u}_A \text{ and } \vec{\rho} = \vec{r}_B + d_B\vec{u}_B. \quad (8)$$

As follows from (8),

$$\vec{r}_A + d_A\vec{u}_A = \vec{r}_B + d_B\vec{u}_B. \quad (9)$$

The expression (9) includes unknown variables d_A and d_B. For the determination of these variables, it is necessary to multiply (9) on $\vec{u}_A$

$$\vec{u}_A r_A + d_A = \vec{u}_A\vec{r}_B + d_B\vec{u}_A\vec{u}_B. \quad (10)$$

Respectively, after multiplication (9) on $\vec{u}_B$ we will obtain

$$\vec{u}_B r_A + d_A\vec{u}_B\vec{u}_A = \vec{u}_B\vec{r}_B + d_B. \quad (11)$$

Based on (10), (11) and introducing the notation $k = \vec{u}_A\cdot\vec{u}_B$, the following set of equations can be derived

$$\begin{aligned} d_A - kd_B &= \vec{s}\vec{u}_A, \\ kd_A - d_B &= \vec{s}\vec{u}_B. \end{aligned} \quad (12)$$

Solving the set of equations (12), we will obtain

$$d_A = \frac{1}{1-k^2}\vec{s}\cdot(\vec{u}_A - k\vec{u}_B). \quad (13)$$

After substitution (13) in (1), the radius vector can be determined in the following way

$$\vec{\rho} = \vec{r}_A + d_A\vec{u}_A = \vec{r}_A + \frac{\vec{s}\cdot(\vec{u}_A - k\vec{u}_B)}{1-k^2}\vec{u}_A. \quad (14)$$

4) Analytical Solution: Case 2

The disadvantage of the considered case 1 is the necessity to determine both the line segment d_A and the line segment d_B, although for solving a problem, it is sufficient to determine only one of them. So, the first solution leads to additional calculating errors.

Consider the technique, which requires determining the line segment d_B only. To avoid the necessity of obtaining the line segment d_A, we will obtain a cross-product of (1) on $\vec{u}_B$. This product can be represented in the following form

$$\vec{r}_A\times\vec{u}_B + d_A\vec{u}_A\times\vec{u}_B = \vec{r}_B\times\vec{u}_B + d_B\vec{u}_B\times\vec{u}_B = \vec{r}_B\times\vec{u}_B. \quad (15)$$

After some transformations in (15), it is possible to write

$$d_A\vec{u}_A\times\vec{u}_B = (\vec{r}_B - \vec{r}_A)\times\vec{u}_B. \quad (16)$$

Based on (16), we will obtain

$$d_A = \frac{|(\vec{r}_B - \vec{r}_A)\times\vec{u}_B|}{|\vec{u}_A\times\vec{u}_B|}. \quad (17)$$

5) Using Arbitrary Lines-of-Sight

We will consider a technique of determining target coordinates based on an arbitrary quantity of lines-of-sight. The location of points A_i can be given by vectors $\vec{r}_i$ (Fig. 2).

The location of the point *M* can be given by the vector $\vec{\rho}$. The location of the point *M* relative to the point A_i can be determined by the vector $\vec{d}_i$. The unit vector of the line-of-sight is denoted $\vec{u}_i$.

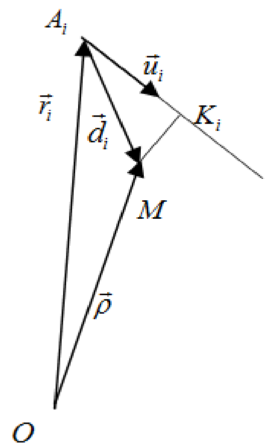

Fig. 2. Geometrical representation of determining target coordinates using arbitrary lines-of-sight

It should be noted that the line-of-sight does not pass through the point *M* due to some error [8]. The sighting error can be characterized by a distance MK_i from the point *M* to the line-of-sight (see Fig. 2)

$$MK_i = |\vec{u}_i\times\vec{d}_i|. \quad (18)$$

Using the least-squares technique, we will search a point *M* from the point of view of minimizing the following loss function

$$g = \sum_{i=1}^{n}\|\vec{u}_i\times\vec{d}_i\|^2. \quad (19)$$

Finally we will obtain

$$\vec{u}_i\times\vec{d}_i = U_i d_i, \quad (20)$$

where

$$U_i = \begin{bmatrix} 0 & -u_{zi} & u_{yi} \\ u_{zi} & 0 & -u_{xi} \\ -u_{yi} & u_{xi} & 0 \end{bmatrix}, \quad d_i = \begin{bmatrix} d_{xi} \\ d_{yi} \\ d_{zi} \end{bmatrix}.$$

Therefore the loss function (19) taking into account (20) becomes

$$g = \sum_{i=1}^{n} \left\| \vec{u}_i \times \vec{d}_i \right\|^2 = \sum_{i=1}^{n} \left\| U_i d_i \right\|^2 = \sum_{i=1}^{n} (U_i d_i)^{\mathrm{T}} (U_i d_i)$$
$$= \sum_{i=1}^{n} d_i^{\mathrm{T}} U_i^T U_i d_i) = \sum_{i=1}^{n} \delta_i^{\mathrm{T}} H_i d_i, \tag{21}$$

where $H_i = U_i^T U_i$. As $\vec{d}_i = \vec{r}_i - \vec{\rho}$, we can write

$$d_i^{\mathrm{T}} H_i d_i = (r_i - \rho)^{\mathrm{T}} H_i (r_i - \rho) = r_i^{\mathrm{T}} H_i r_i - \rho^{\mathrm{T}} H_i r_i$$
$$- r_i^{\mathrm{T}} H_i \rho_i + \rho^{\mathrm{T}} H_i \rho_i. \tag{22}$$

Taking into consideration the condition $\partial g / \partial \rho = 0$, we can obtain the equality

$$\left(\sum_{i=1}^{n} H_i \right) \rho = \sum_{i=1}^{n} H_i r_i. \tag{23}$$

As follows from (23),

$$\rho = \left(\sum_{i=1}^{n} H_i \right)^{-1} \sum_{i=1}^{n} H_i r_i. \tag{24}$$

To assess the accuracy of the proposed algorithms, let us perform modeling.

We will believe that coordinates of a point M are determined. Coordinates of this point are given by the radius vector $r_M = [12 \;\; 26 \;\; 128]^{\mathrm{T}}$ m. Respectively, points of sighting are given by radius vectors $r_A = [1 \;\; 2 \;\; 3]^T$ m; $r_B = [4 \;\; -15 \;\; 10]^T$ m; and $r_C = [4 \;\; 10 \;\; 10]^T$ m.

An error of sighting vector, for example, $u_A = [u_{Ax} \; u_{Ay} \; u_{Az}]^{\mathrm{T}}$, can be given by introducing coefficients n_x, n_y, n_z close to 1

$$u_A = \begin{bmatrix} n_x u_{Ax} & n_y u_{Ay} & n_z u_{Az} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} n_x & 0 & 0 \\ 0 & n_y & 0 \\ 0 & 0 & n_z \end{bmatrix} \begin{bmatrix} u_{Ax} \\ u_{Ay} \\ u_{Ay} \end{bmatrix} = \begin{bmatrix} n_x & 0 & 0 \\ 0 & n_y & 0 \\ 0 & 0 & n_z \end{bmatrix} \begin{bmatrix} u_{Ax} & u_{Ay} & u_{Ay} \end{bmatrix}^{\mathrm{T}}. \tag{25}$$

Every coefficient in (25) has been calculated as $5 \cdot 10^{-4}\,\mathrm{rand}([-10,10])$. Errors of calculating target coordinates for 10 realizations are represented in Fig. 3. They were calculated using the formula (14). The same calculations carried out by the formula (24) are shown in Fig. 4. Figures 3, 4 show that accuracy of calculations by formula (14) and (24) are equivalent. So, it is convenient to use the algorithm based on the least-squares technique [10].

Figure 5 shows the results of calculation by formula (24) with target coordinates averaging.

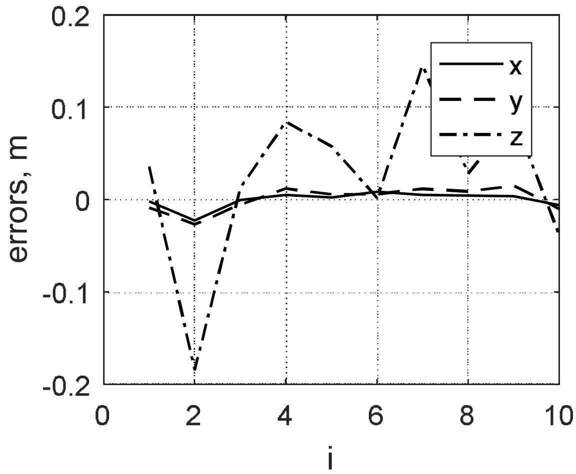

Fig. 3. Errors of calculating coordinates of the target using formula (14)

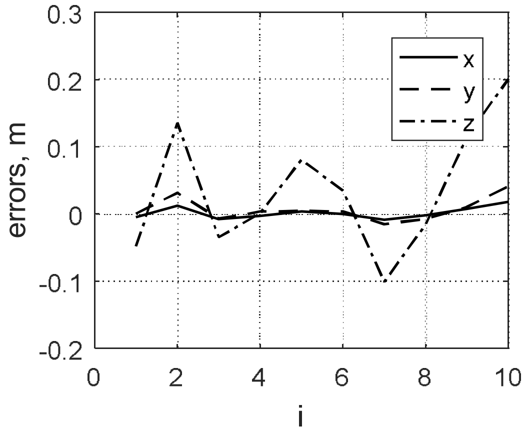

Fig. 4. Errors of calculating coordinates of the target using formula (24)

For target positioning can be used the algorithm based on distances measurements [9]. Suppose we know distances between target M and some points A_j ($i \geq 4$) with known coordinates (Fig. 6). The position of points A_i is given by the vectors $\vec{r}_i$. The position of target M with unknown coordinates is given by the vectors $\vec{\rho}$. The position of the point A_i relative to the point M is given by the vector $\vec{d}_i$.

The solution of problem is given by the formula

$$\rho = \mathbf{K}^{-1} \mathbf{h}, \tag{26}$$

where

$$\mathbf{K} = 2\begin{bmatrix} (r_2 - r_1)^T \\ (r_3 - r_1)^T \\ \vdots \\ (r_n - r_1)^T \end{bmatrix}, \quad \mathbf{h} = \begin{bmatrix} d_1^2 - d_2^2 + r_2^2 - r_1^2 \\ d_1^2 - d_3^2 + r_3^2 - r_1^2 \\ \vdots \\ d_1^2 - d_n^2 + r_n^2 - r_1^2 \end{bmatrix}.$$

For simulation to three point with radius vectors r_A, r_B, r_C, the 4th point A_D was added. This point has the radius vector $r_D = [1 \ 40 \ -7]^{\mathrm{T}}$.

For measurement errors, a normal distribution law has been adopted, namely, taken

$$d_i = \|\vec{r}_i - \vec{\rho}\| \left(1 + 1 \cdot 10^{-4} \,\mathrm{rand}\, n\right).$$

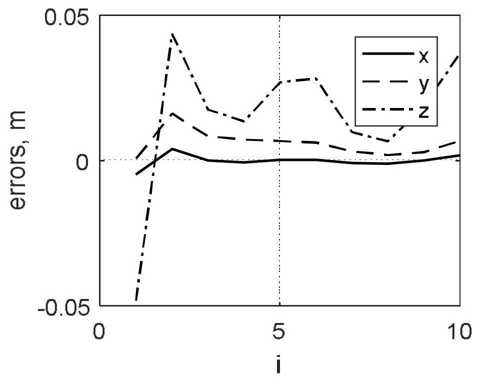

Fig. 5. Calculating errors with averaging

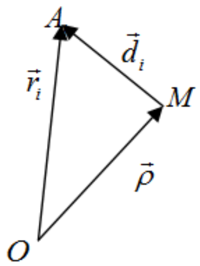

Fig. 6. The points and the vectors

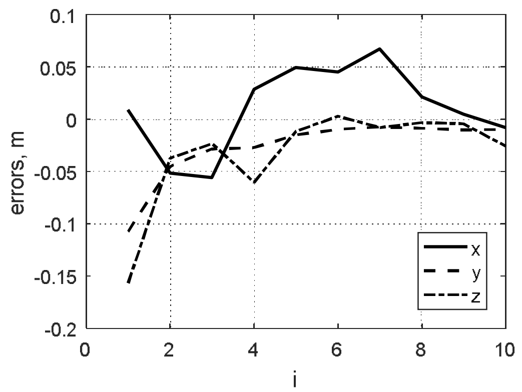

Fig. 7. Errors of calculating coordinates of the target using formula (26)

IV. Conclusions

Some algorithms of target coordinates determining are proposed. Simulation resuls prove the sufficiently high accuracy of calculating coordinates of a target. For solving the problem, it is convenient to use the least-squares technique. The accuracy improves with the increase of a distance between sighting points. The same effect is observed in the case of measurement averaging. Proposed algorithms can also be used for solving the inverse problem when it is necessary to determine UAV coordinates based on lines-of-sight or distances to the known points.

The main differences in the sampling of models of the stabilization system at the stages of synthesis and analysis of the obtained results are represented.

The effectiveness of the proposed approaches is confirmed by the simulation results.

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Л. М. Рижков, О. А. Сущенко. Про деякі алгоритми позиціонування цілі

У статті розглядається проблема визначення координат положення цілі на основі використання декількох ліній візування, отриманих за допомогою антен, що встановлюються на безпілотних літальних апаратах. Запропоновано алгоритми позиціонування цілі за допомогою безпілотних літальних апаратів. Запропоновані алгоритми вимагають знання координат кількох положень безпілотного літального апарату. Для вирішення задачі використовується техніка найменших квадратів. Розглядаються деякі шляхи вирішення проблеми. У першому випадку алгоритми передбачають знаходження декількох прямих ліній візування цілі. Далі для вирішення проблеми використовується геометричний або аналітичний метод. У другому випадку алгоритм передбачає вимірювання відстаней від рухомого об'єкта до цілі. Завдання вирішуються у векторній та векторно-матричній формі. Представлені результати моделювання, що підтверджують ефективність запропонованих алгоритмів. Отримані результати можуть бути корисними як для цивільних, так і для спеціальних областей застосування.

Ключові слова: алгоритм; аналітичне та геометричне рішення; лінія візування; позиціонування цілі; безпілотні літальні апарати.

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Л. М. Рижков, О. А. Сущенко. Об алгоритмах позиционирования цели

В статье рассматривается проблема определения координат положения цели на основе использования нескольких линий визирования, полученных с помощью антенн, устанавливаемых на беспилотных летательных аппаратах. Предложены алгоритмы позиционирования цели с помощью беспилотных летательных аппаратов. Предложенные алгоритмы требуют знания координат нескольких положений беспилотного летательного аппарата. Для решения задачи используется техника наименьших квадратов. Рассматриваются некоторые пути решения проблемы. В первом случае алгоритмы предусматривают нахождение нескольких линий визирования цели. Далее для решения проблемы используется геометрический или аналитический метод. Во втором случае алгоритм предусматривает измерение расстояний от подвижного объекта к цели. Задачи решаются в векторной и векторно-матричной форме. Представлены результаты моделирования, подтверждающие эффективность

предложенных алгоритмов. Полученные результаты могут быть полезными как для гражданских, так и для специальных областей применения.
Ключевые слова: алгоритм; аналитическое и геометрическое решение; линия визирования; позиционирование цели; беспилотные летательные аппараты.

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