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MATHEMATICAL MODEL OF DYNAMICALLY TUNED GYROSCOPE

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Abstract—*The paper deals with creation of a set of models of the dynamically tuned gyroscope by means of Simulink. The developed model is close to the real device as it includes models of both the gyroscope and service electronic devices. Models of angle and rate meauring devices based on the dynamically tuned gyroscope are obtained. Simulation of measuring processes had been carried out. The possibilities of small angular rate measurement were studied. The influence of sampling intervals on accuracy of measuring processes was researched. The obtained results can be useful for creation of navigation systems used in high-accurate autonomous applications.*

Index Terms—Inertial sensor; dynamically tuned gyroscope; rate gyroscope; simulation; sampling interval.

I. INTRODUCTION

Inertial sensors are important components for a number of practical applications. Robotics, autonomous vehicles, airplanes, ships, satellites and inertially stabilized platforms are some examples of such applications. They make us to realize the importance for many countries of having their own development of inertial technology [1].

The dynamically tuned gyroscopes (DTG) can be used in the inertially stabilized platforms as measuring instruments of servo systems [2]. Such platforms are widely used in aviation and rocket technologies to solve vehicle navigation and also stabilization of special devices including direction finders, viewfinders, objectives of photo cameras and so on [3].

Particularly, the gyroscopes usually involve high cost manufacturing processes, sophisticated technology and expertise, so that the necessary performance on vehicle navigation or device positioning can be achieved. The dynamically tuned gyroscopes are one of the options nowadays that can provide the required performance for many applications at a reasonable price. They demand considerable less investment in materials, machinery and man power when compared to the other usual sensors such as the floated or ring-laser gyroscopes [1].

Dynamically tuned gyros can be designed as three-degree-of-freedom gyroscopes, which can measure attitude of a vehicle. Such a device supplemented feedback can be used for measurement of vehicle angular rates. From the technical point of view it means that feedback amplifier must be included in the control contour. Thus, considered device can operate in two modes. This provides definite advantages for using above stated DNGs in high-accurate navigation and stabilization systems.

II. PROBLEM STATEMENT

The problem is development of the mathematical model of DTG in all possible modes of its operation. Such specific problems as influence of sampling interval and cross connections on the quality indices of transient processes will be studied. The structural scheme of the measuring device model based on DTG is given in Fig. 1. Such a device can be used for measurement of angles, which determine position of a vehicle in the inertial space. Depending on location of measuring axes, the device can carry out functions of the vertical or directional gyroscope.

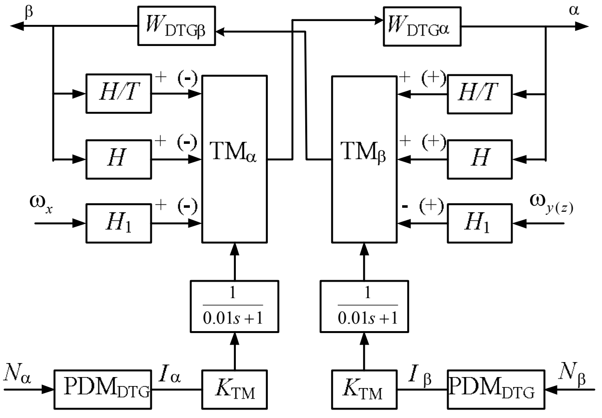

Fig. 1. Structural scheme of a measuring device based on DTG (vertical gyroscope): α, β are measured angles; $W_{DTG\alpha}$, $W_{DTG\beta}$ are transfer functions by angles α, β; H is the kinetic moment; T is the time constant; ω_x; $\omega_{y(z)}$ are input angular rates; $H_1 = 0.999H$; TM denotes torque motor; PDM denotes pulse duration modulator; N_α, N_β are control codes, which are determined in the computing device; I_α, I_β are control currents, which enter to PDM inputs

The model represented in Fig. 1 corresponds to the vertical gyroscope. Notations in Fig. 1, which are given in brackets, correspond to the directional gyroscope.

Thus, the represented device includes DTG, torque motors and pulse duration modulators. To use the above represented device as a rate gyroscope is possible using feedback by output signals. The structural scheme of such a device is represented in Fig. 2.

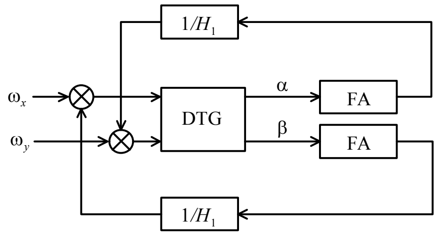

Fig. 2. Rate gyroscope based on DTG: FA is feedback amplifier

The rate gyroscope based on DTG is the high-accurate measuring instrument. Therefore it can be used in the high-precision inertial navigations systems and attitude and heading reference systems (for example for the marine navigation), where setting to horizon can be done based on measurement of the Earth angular rates.

The purpose of the paper is development of models in modes of attitude and angular rate measurement. The models will be created by means of Simulink. Including service electronics models makes models close to real systems. This allows study important design problems including influence of sampling interval and cross-connections on measurement accuracy.

III. Analysis Of Last Researches And Publications

There are many textbooks and papers, which deal with features of operation and simulation of measuring instruments based on DTG [1], [2]. Applications of DTGs are considered in [3]. Detailed mathematical description of DTG with the gas dynamic bearing is represented in [4].

Above stated applications are extended nowadays. And it is important to improve mathematical models of DTG in order to satisfy new rigid requirements to accuracy, extension of functional possibilities and so on.

IV. Mathematical Model Of Attitude Dynamically Tuned Gyroscope

Firstly, the mathematical model of the measuring instrument based on DTG with orientation of the kinetic moment providing measurement of heading will be considered. Such a device corresponds to the directional gyroscope. In accordance with [5] the mathematical model of DTG represents a set of the differential equations

$$\left.\begin{aligned} &J\ddot{\alpha} + d\dot{\alpha} + H\dot{\beta} + \frac{H}{T}\beta + c\alpha = -H_1\omega_x + M_z;\\ &J\ddot{\beta} + d\dot{\beta} - H\dot{\alpha} - \frac{H}{T}\alpha + c\beta = H_1\omega_z - M_x, \end{aligned}\right\}; \qquad (1)$$

here J is a sum of equatorial moments of the rotor and gimbals respectively; d is the damping coefficient; H is the gyroscope kinetic moment; T is the gyroscope time constant; c is the residual rigidity of gimbals; $H_1 = H(1-s)$; $s = 10^{-3}$; ω_x, ω_z are projections of the platform angular rates onto the platform axes; M_x, M_z are external disturbance moments.

The model of the vertical gyroscope can be easy obtained based on (1) and structural scheme represented in Fig. 1. It is not given in the paper due to its similarity to the directional gyro model.

The Simulink model can be created by means of direct implementation. But such a model is too difficult for both development and understanding. Therefore it is convenient to develop the model using representation by means of transfer functions. For this it is possible to use the Laplace transform. Neglecting external moments we will represent the model (1) in the following form

$$\begin{aligned} &Jp^2\alpha + dp\alpha + Hp\beta + \frac{H}{T}\beta + c\alpha = -H_1\omega_x;\\ &Jp^2\beta + dp\beta - Hp\alpha - \frac{H}{T}\alpha + c\beta = H_1\omega_z, \end{aligned} \qquad (2)$$

where p is the Laplace operator.

The set equations (2) can be solved relative to the output variables

$$\begin{aligned} \alpha &= \frac{1}{Jp^2 + dp + c}\left[-\left(H + \frac{H}{T}\right)\beta - H_1\omega_x\right];\\ \beta &= \frac{1}{Jp^2 + dp + c}\left[\left(H + \frac{H}{T}\right)\alpha + H_1\omega_z\right]. \end{aligned} \qquad (3)$$

To provide effective functioning of the developed model it is convenient to divide and multiply the equations (3) by p

$$\begin{aligned} \alpha &= \frac{p}{Jp^2 + dp + c}\left[-H\beta - \frac{H}{Tp}\beta - \frac{H_1}{p}\omega_x\right];\\ \beta &= \frac{p}{Jp^2 + dp + c}\left[H\alpha + \frac{H}{Tp}\alpha + \frac{H_1}{p}\omega_z\right]. \end{aligned} \qquad (4)$$

Due to these transformations the transfer functions of (4) become proper. Using equations (4) it is easy to create the Simulink model of DTG based on transfer functions. Such a model is characterized by the high speed of operation during simulation. This is important for research of transient processes of navigation systems.

Elimination of cross connections between measuring channels can be achieved by means of ideal dynamical tuning [5]. But such a situation is impossible in practice. The approach to elimination of cross connections between measuring channels is given in [6]. It is based on using two dynamically tuned gyroscopes, kinetic moments of which are directed in opposite directions. Really, based on equations (4) it is possible to derive expressions for determination of angle α

$$\alpha_{+} = \frac{-H_1\omega_x(Jp^2+dp+c) - H_1\omega_z(Hp+H/T)}{(Jp^2+dp+c)^2+(Hp+H/T)^2}. \quad (5)$$

The appropriate expression for the gyroscope with the kinetic moment of the opposite sign becomes

$$\alpha_{-} = \frac{-H_1\omega_x(Jp^2+dp+c) + H_1\omega_z(Hp+H/T)}{(Jp^2+dp+c)^2+(Hp+H/T)^2}. \quad (6)$$

As follows from (5), (6) it is possible to eliminate cross connections of DTG measuring channels. The mathematical model of the device based on two DTG with oppositely directed kinetic moments is represented in Fig. 3. The model of the rate gyroscope based on DTG including feedback is represented in Figs 4 and 5.

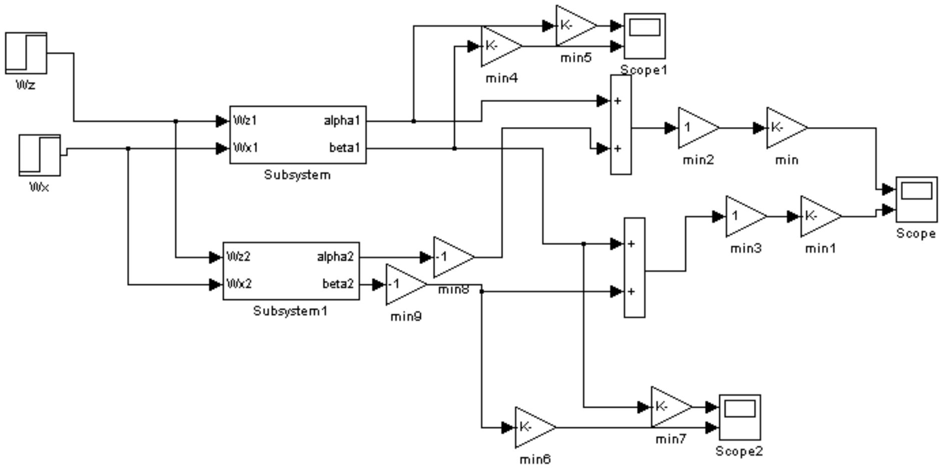

Fig. 3. Simulink-model of the measuring device based on two DTG

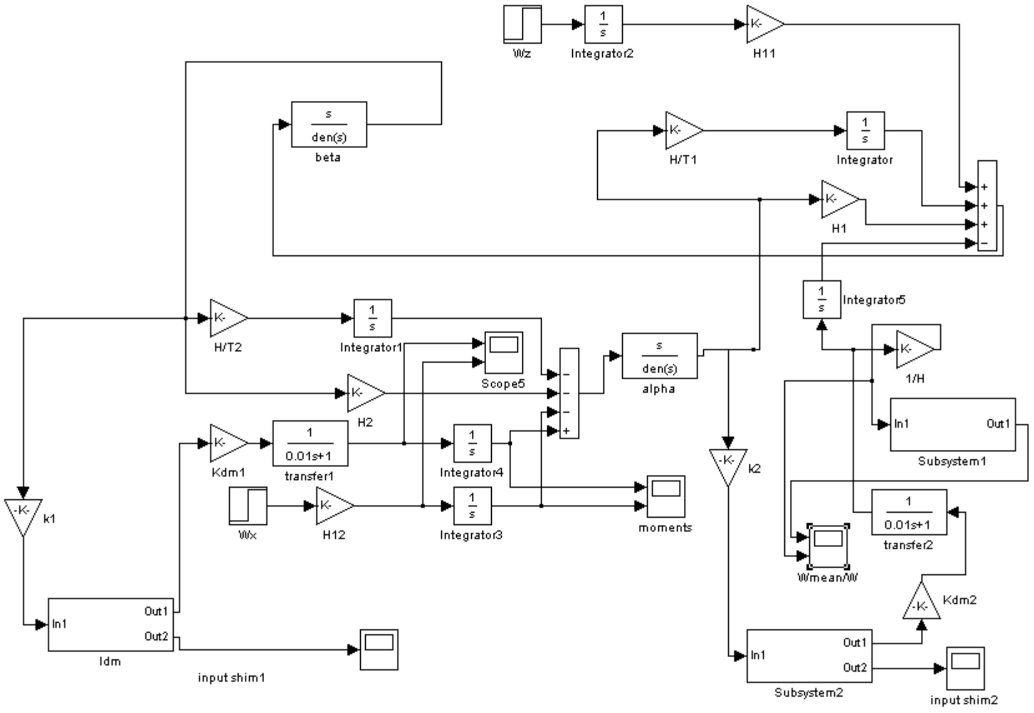

Fig. 4. Simulink-model of the rate gyroscope based on DTG

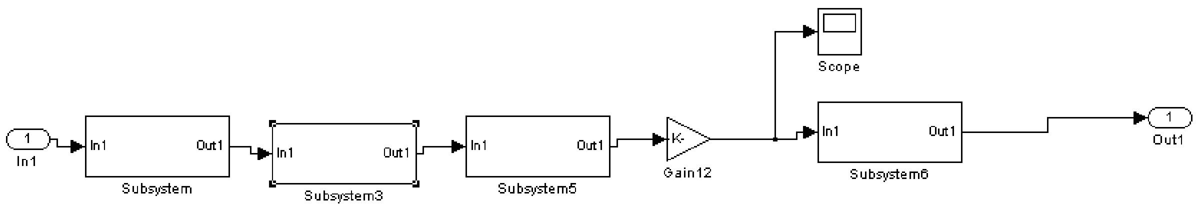

Fig. 5. Structure of feedback

Specific feature of the rate gyroscope based on DTG is usage of the feedback. In this case the mathematical model of the measuring instrument must be supplemented by the models of electronic units, including amplifier, digital filter, analog-digital converter, and pulse width modulator. Mathematical models of units, which form feedback of the device, are represented in Figs 6 – 9.

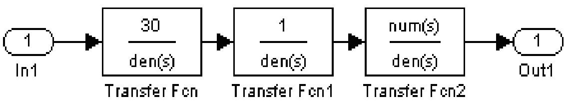

Fig. 6. Amplifier

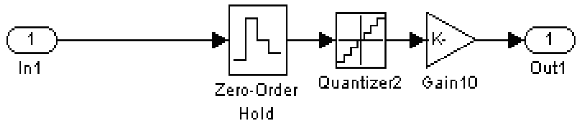

Fig. 7. Analog-digital converter

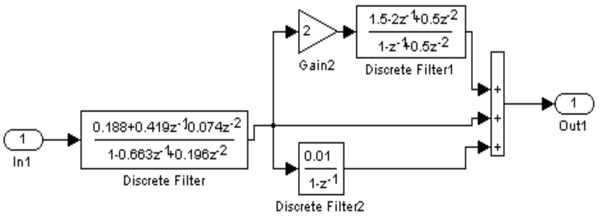

Fig. 8. Digital filter

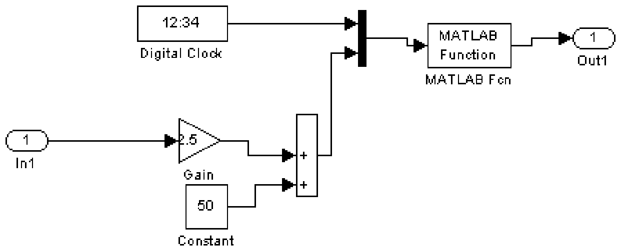

Fig. 9. Pulse-width modulator

The represented models correspond to the real devices as far as possible.

V. SIMULATION RESULTS

It should be noted that time averaging had been used for representation of simulation results. The appropriate model is given in Fig. 10. Results of simulation of small angular rate measurement are given in Fig. 11a, 11b.

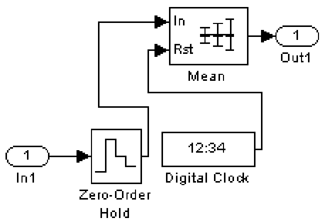

Fig. 10. Averaging of simulation results

The transient process of small angular rate measurement (0.0007 rad/s) is given in Fig. 11a. Sampling of signal processing is 1 ms. Figure 11b represents the same results after averaging. Thus, simulation results prove the possibility to measure small angular rates by means of DTG. Such results prove also the possibility to use above mentioned device for setting to the initial meridian during heading determination. (Rate values are close to the Earth angular rate) [6]. It is known that sampling interval influences on accuracy and reliability of information processing. This defines quality of control transient processes. Therefore research of sampling interval value on transient process characteristics was one of purposes of simulation using the developed DNG model. To solve this problem, different combinations of digital capacities of the analog-digital converter (ADC), computing device (CD) and pulse duration modulator (PDM) were used.

TABLE I

COMPARISON OF SAMPLING INTERVALS

# variant	Digital capacity of ADC,s	Digital capacity of CD, s	Digital capacity of PDM, s
1	0.0012	0.0012	0.0012
2	0.0048	0.0048	0.0048
3	0.0012	0.0048	0.0048
4	0.0012	0.0012	0.0048
5	0.0048	0.0012	0.0012
6	0.0048	0.0048	0.0012
7	0.0012	0.0048	0.0012
8	0.0048	0.0012	0.0048

Simulation results are given in Fig. 11c, 11d. Output signals of PDM are given here. Simulation time was accepted to be equal to 1 s. The reference signal is equal to 0.2 arcmin.

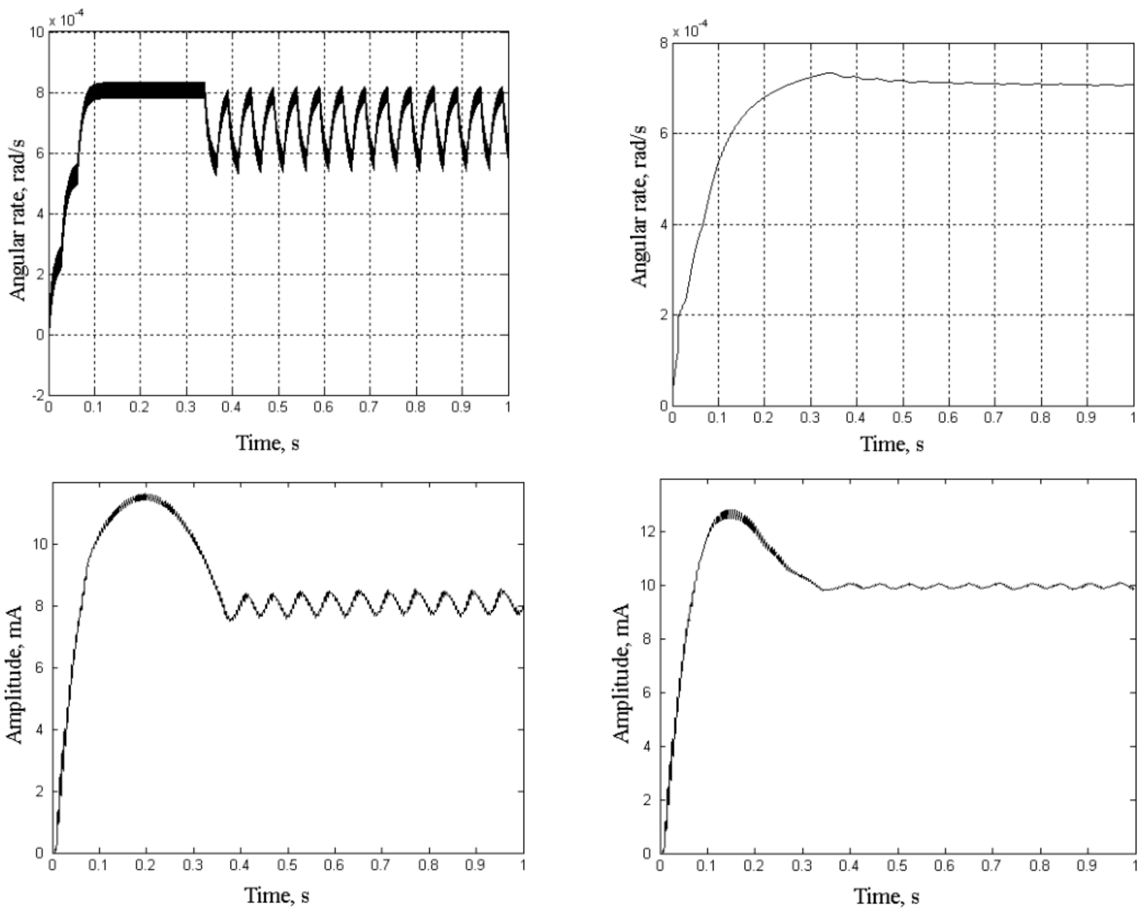

Fig. 11. Simulation results: (a) corresponds to the small angular rate (0.0007 rad/s), (b) corresponds to the average small angular rate; c, d correspond to variants of different sampling intervals (variants 4 and 8 of Table I)

VI. CONCLUSIONS

The dynamically tuned gyro model based on Simulink is created. This allows us to take into consideration features inherent to real systems and provides simulation close to real system functioning.

Models of DTG in modes of measurement of both attitude and angular rates are developed.

Simulation results are given and analysed. The possibility to measure small angular rates is proved. Influence of sampling intervals is researched.

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О. А. Сущенко, Ю. В. Белявцев Математична модель динамічно налаштованого гіроскопа

Представлено низку моделей динамічно налаштованого гіроскопа, створених у середовищі Simulink. Розроблена модель наближається до реального приладу, оскільки вона містить моделі як гіроскопа, так і сервісної електроніки. Розглянуто моделі вимірювачів кутів та кутових швидкостей на основі динамічно настроюваного гіроскопа. Виконано моделювання вимірювальних процесів. Досліджено можливості вимірювання малих кутових швидкостей. Отримані результати можуть буди корисні під час створення навігаційних систем для високоточних та автономних застосувань.

Ключові слова: інерціальний датчик, динамічно налаштований гіроскоп, швидкісний гіроскоп, моделювання, інтервал дискретизації.

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О. А. Сущенко, Ю. В. Белявцев Математическая модель динамически настраиваемого гироскопа

Представлен ряд моделей динамически настраиваемого гироскопа созданных в среде Simulink. Разработанная модель приближена к реальному прибору, поскольку включает модели, как гироскопа, так и сервисной электроники. Рассмотрены модели измерителей углов и угловых скоростей на основе динамически настраиваемых гироскопов. Выполнено моделирование измерительных процессов. Исследованы возможности измерения малых угловых скоростей. Полученные результаты могут быть полезны при создании навигационных систем для высокоточных и автономных применений.

Ключевые слова: инерциальный датчик; динамически настраиваемый гироскоп; скоростной гироскоп; моделирование; интервал дискретизации.

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