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V. M. Sineglazov

Aeroelastic Vibrations Calculation for Wind Power Plants Rotors

Aviation Computer-Integrated Complexes Department, National Aviation University, Kyiv, Ukraine
E-mail: svm@nau.edu.ua

Abstract—The problem of wind turbines aeroelastic rotor vibrations calculation is considered. It shows that the analysis of the elastic blade aero stability in this case differs from the classical approach.

Index terms—Aeroelastic vibrations; wind power plant; rotor; design.

I. INTRODUCTION

The vibrations analysis is very important during power plant rotor design.

Occurrence of the vibrations in the elements of the wind power plant with a vertical pivot is not so far fully examined due to a number of circumstances. The vibration units cannot be eliminated completely. As a consequence, possibilities of reducing vibration level in various ways are being researched.

II. PROBLEM STATEMENT

Based on assumption that the main attention should be paid to reduce vibration, rotor design process actually splits into a consistent and repetitive (iterative) carrying out two theoretical and one experimental procedures with further analysis of wind turbine parameters on satisfactory compliance with standards and regulations. Theoretical calculations are as follows:

– on the basis of a three-dimensional mathematical model, built in Ansys, Nastran or another computer program (or imported from SolidWorks, Compas), determine the properties of the researched wind turbines components with finite element method, calculate the proper frequencies of their vibrations, optimize rotor configuration, carry out the selection of acceptable materials to maximize vibration reduction;

– on the basis of functional mathematical model developed in the VisSim, MatLab or similar visual function simulator, analyze the impact of external aerodynamic and centrifugal forces and moments on the rotor, identify possible resonances and carry out design optimization by controlling stiffness of the elements to shift the resonant phenomena to safe frequency range. It this article the aeroelastic vibrations calculation will be done.

III. INVESTIGATION PROCEDURE

The sequence of theoretical studies can be represented as shown in Fig. 1. Mathematical models can be built using other software packages, in a similar way.

Model construction and investigation is always done with a number of assumptions that do not have a serious impact on the end result (laminarity and stability of wind flow in terms of pressure and direction, uniformity of materials, etc.). During the rotor research it is taken to be that while wind power plant is working, its rotor from the start while freewheeling passes through the whole range of vibrations from zero to maximum frequency determined by the manufacturer. When connecting the load to the generator vibration impact of wind turbines on the foundation is reduced [6], [7], [10]–[12] by reducing the amplitude of the vibrations.

Each wind power plant element has its own oscillation frequency, backlash and intercommunication with other elements.

A detailed consideration of the nature of vibration occurrence in a real wind power plant allows to see that while wind turbine is working vibrations occur which are transmitted to foundation (structures on which it is installed), and air environment. The result is a complex of vibrations and generated by these sound oscillations (sound, infrasound, ultrasound).

The amplitudes and frequencies of wind turbine components joint oscillations and constructions are substantially dependent on the design parameters of the wind turbine and construction on which the wind turbine is installed. Their oscillations can only be regarded as joint oscillations.

The main factors causing wind turbine vibrations which are transmitted to the foundation are:

– periodic inertial forces caused by the displacement of the rotor center of mass from the axis of rotation;

– periodic inertia moments caused by occurrence of non-zero centrifugal inertia moments of rotor;

– aerodynamic forces acting on the windwheel rotor;

– aerodynamic moments acting on the rotor propeller.
The frequency of the inertial forces and moments is equal to rotation frequency of windwheel. The dependence of these forces and moments from time has the character of harmonic oscillations.

The oscillations frequency of the aerodynamic forces is $N$ times larger than the rotor speed ($N$ is the number of rotor blades). But the nature of the actual vibrations is different from harmonic. Therefore, they can be represented as the sum of harmonic oscillations with frequencies of $N^n$, $2N^n$, $3N^n$, $4N^n$.

The oscillation frequency of aerodynamic moments is $n$ times larger than rotor rotation frequency ($n$ is the number of blades in one section of two-part rotor). Nature of moment oscillations is different from harmonic too. Therefore, they can be represented as the sum of harmonic oscillations with frequencies of $n^v$, $2n^v$, $3n^v$, $4n^v$. Rotor rotation frequency $v_s$ is constantly changing, along with it frequencies of inertial and aerodynamic forces and moments are also changing. In case when the proper wind turbine frequency coincides with one of mentioned frequencies, and the strength amplitude corresponding to this frequency being sufficiently high, an occurrence of resonance would be possible for some time. The degree of exposure of wind turbine on the foundation at resonance multiple increases.

The parameters of wind turbine vibration exposure of wind turbines can be determined either by calculation way using a mathematical model of the wind turbine, or by measurements in bench tests of experimental. The most rational is the combination of these two ways, when the parameters of the mathematical model are refined by the results of the measurements, then the accuracy of the mathematical modeling increases and it is possible to obtain data for the modes of operation of the wind turbine, which were not implemented in full-scale tests, for example, due to lack of sufficient wind.

Under the characteristics of vibration exposure is supposed a set of amplitudes and frequencies of harmonic oscillations, the sum of which coincides to the actually measured relation from exposure force time on the foundation. These characteristics can be obtained by decomposition of measured or calculated force relation in a Fourier series.

IV. AEREOELASTIC VIBRATIONS OF THE ROTOR

Under the influence of aerodynamic loads mentioned above, oscillations of blades and cross-arms occur. In general, they are joint transverse (flexural in three planes – horizontal and two vertical perpendicular) and torsional oscillations [1], [2], [5].

The usual approach to the study of forced oscillations is to put on the blades of the rotor transverse loads $Y_s$, $M_s$ as functions which are explicitly dependent on time. Thus for each ith blade it is influenced the load with its own phase.

When solving corresponding differential equations of cross bending and twisting of the blade it is necessary to take into account the centrifugal
forces [9]. If the transverse deformations of the blade are small, then the change in distance from axis of rotation to the center of mass of the blade sections can also be ignored. In this case, the overall load on the blade (considering only transverse oscillations) will be represented by the sum of constant centrifugal and variable aerodynamic force. With these parameter values of rotor the centrifugal force \( P_c \) related to the area of the lateral projection of the blade is

\[
P_c \frac{m \omega^2 R b L}{2 b L} = \frac{1}{2} m \omega^2 R,
\]

(1)

where \( L \) is the blade length; \( m \) is the linear blade weight; \( R \) is the ratio of rotor radius to half chord of the blade.

To calculate the stress acting in blades and cross-arms of rotor, simplified view of one of the rotor sections in the form of statically indeterminate double frame shown in (Fig. 2a) can be considered. It should be kept in mind that the cross-arm hardness varies depending on the angular velocity \( \omega \). Indeed, in addition to the load at the point where cross-arm is attached to the blade, in each of the intersection, which is located at a distance \( S \) from the axis of the rotor, the centrifugal force acts being

\[
N(s) = \int_{S}^{R_b} m_{cross} \omega^2 S dr,
\]

(2)

where \( m_{cross} \) is the linear mass of the cross-arm.

Since the load is constant in time, it can be taken into account when determining the flexural rigidity of the cross-arm.

Aerodynamic and centrifugal loads are usually situated in the plane which is perpendicular to the rotor rotation axis [3], [4]. Therefore, under the investigation of aeroelastic vibrations it can be limited, perhaps, by consideration of only one symmetric oscillation shape shown in (Fig. 2, a) by dotted line.

If take into account the influence of cross-arm’s, then the scheme, shown in Fig. 2, b is right. In this case, the predicted oscillation shape (taking cross-arms own weight into account) will be different (dotted line in Fig. 2, b).

Since the front aerodynamic center, in which aerodynamic forces are applied is not always situated at 1/4 of the chord of blade profile sock and slips along the chord, and may even be “behind” the middle of the profile, it is necessary to take into account moments occurring in this case with regard to the point of the blade is attached to cross-arm.

This situation occurs when there are disruptions of the flow from the profile (rotor start and stop modes, wind gusts). Furthermore, if we accept fully justified assumption that the pulling forces do not always tend to spin the rotor with appropriate speed, oscillations of “blade – cross-arms – roll” system are to be considered with regard to constant with some medium rotation speed.

Thus, in general, study of forced oscillations of the rotor frame construction is a complex task considering to the joint flexural (blade, cross-arm) and torsional (blade) rotor elements oscillations. On the other hand, since the blade’s line of centers of mass passes through the junction points of the blade and cross-arm, flexural and torsional vibrations turn out to be unrelated and their separate study is possible.

The problem practical solution about forced oscillation of elements’ spinning rotor reduces to the
determination of dynamic loads and rotor speeds in steady mode; then determined loads are placed into the right side of equations which describe own flexural and torsional oscillations of the considered frame construction. The solution of these equations can be found using one of the approximate numerical methods (finite difference method, FEM, Bubnov–Galerkin method). But it can already be said that if the first natural frequency of oscillation of the rotor frame is significantly higher than the prevailing (first) aerodynamic load frequency, the displacement (and voltage) of the rotor elements under the purely forced oscillations would be insignificant. At the same time, forced oscillations in the first eigenform with large amplitude can be excited under the influence of higher harmonics of the aerodynamic forces.

Then the issue arises of the possibility of aeroelastic instability occurring and therefore occurring auto-oscillation of rotor blade flutters phenomenon associated with this instability. However, as shown above, the flow which surrounds the blade is not uniform: it is changing periodically in terms of the speed as well as the direction. Therefore, in this case, flutter in its pure form cannot be considered, is it only if we imagine the rapid rotation of the rotor in calm weather. At the same time, the issue of loss of aeroelastic stability of the blade can be set in this case. Moreover, since the blade lost stability is to a much greater extent susceptible to variable aerodynamic loads, than the stable one, this issue is becoming the one of particular relevance.

Aeroelastic stability analysis of the blade in this case differs from the traditional approach (that is, given the flow is uniform) and involves investigation of forced oscillations equations of horizontal blade section inside the space between cross-arms. In this particular intersection the joint flexural-torsional oscillations are possible. After the constant rotation frequency is determined, the disturbed motion of the blade sections can be viewed in the rotating coordinate system X0Y relate, for example, with the point of intersection of the elastic axis cross-arms and blades in the unperturbed motion. The corresponding scheme (Fig. 3) given for the general case of the relative location of the center from the rigidity, weight of the M section, the point O of the blade attachment to the cross member and front aerodynamic center of A.

\[
V_0^2 = \left[V_x(t) - x_0\right]^2 + (-y_0)^2.
\]

In this case, the aerodynamic forces \(X_0\) and \(Y_0\) that influence on the profile of unit length blade which is rotating and oscillating can be written as:

\[
V^0 = V_x(t) + V_y(\dot{x}_0, x_0, \dot{\varphi}_0),
\]

where

\[
V_0 = V_x(t) + V_y(\dot{x}_0, x_0, \dot{\varphi}_0)
\]
\[
X_0 = \rho b Y_0^2 \left[ C_y^f (\alpha_t, \dot{\alpha}_t) \cos \Delta \phi_t \right. \\
- C_y^f (\alpha_t, \dot{\alpha}_t) \sin \Delta \phi_t \bigg],
\]
(4)

\[
Y_0 = \rho b Y_0^2 \left[ C_y^f (\alpha_t, \dot{\alpha}_t) \sin \Delta \phi_t \right. \\
+ C_y^f (\alpha_t, \dot{\alpha}_t) \cos \Delta \phi_t \bigg],
\]
(5)

where \( \Delta \phi_t = \Delta \phi + \Delta \phi_0 \).

Not difficult to see that when \( \phi_0 = \dot{y}_0 = \dot{x}_0 = 0, \) expressions (4) are described the external purely forced load. Let’s take profile oscillations equations in the simplest form (see Fig. 3).

\[
\dot{y}_0 + \epsilon_y \dot{y}_0 + \omega_y^2 y_0 - x_m \ddot{\phi}_0 = Y_0,
\]

\[
\dot{x}_0 + \epsilon_x \dot{x}_0 + \omega_x^2 x_0 = X_0,
\]

\[
\ddot{\phi}_0 + \epsilon_{\phi} \ddot{\phi}_0 + \omega_{\phi}^2 \phi_0 - x_m \dot{y}_0 = (r_b - x_c) Y_0.
\]

Equation (6) has been deduced assuming angle \( \alpha_0 \) is small. This assumption also allows to suppose a linear dependency of aerodynamic forces from angle \( \alpha_t, \Delta \phi_0 \) since angles \( \alpha_t \) are also considered to be small. Then instead of (2) we can write

\[
\alpha_0 = \phi_0 - \frac{\dot{y}_0}{V_0 - \dot{x}_0},
\]
(7)

and instead of (4) consider the expressions for \( X_0, Y_0 \) (in quasi-stationary approximation):

\[
X_0 = \rho b V_t^2 \frac{\partial C_y}{\partial \alpha} \alpha, \quad Y_0 = \rho b V_t^2 \frac{\partial C_y}{\partial \alpha} \phi_0.
\]
(8)

Later on we will not consider the second equation of the system (6) since it is not related to the others (if \( X \) is neglected in (7)) and has a little impact on the value of critical flutter speed. In this case equation (6) with (8) taken into account will take the following form:

\[
\dot{y}_0 + \epsilon_y \dot{y}_0 + \omega_y^2 y_0 - x_m \ddot{\phi}_0 = \rho b V_t^2 \frac{\partial C_y}{\partial \alpha} \phi_0 - \frac{\dot{y}_0}{V_t}.
\]

\[
\ddot{\phi}_0 + \epsilon_{\phi} \ddot{\phi}_0 + \omega_{\phi}^2 \phi_0 - x_m \dot{y}_0 = (r_b - x_c) \rho b V_t^2 \frac{\partial C_y}{\partial \alpha} \phi_0 - \frac{\dot{y}_0}{V_t}.
\]
(9)

Apparently, the system (9) is a system with periodic coefficients like \( V_t(t) \) and \( Y_0(t) \). The external load is determined by terms which contain \( V_t(t) \alpha(t) \).

Occurrence of periodic factors at \( \phi_0 \) and \( \dot{y}_0 \) in the right sides of the system (9) reduces the flutter problem to the problem of dynamic stability of parametrically excited system. This problem is very different from the classic, in which wing resistance in a uniform flow is investigated, both by the results (critical values of parameters) and by the complexity of their acquisition. Regarding this we can only note that the results of research on wing resistance in non-uniform flow are unknown to us. At the same time, occurrence of a periodic component of the flow rate, apparently, could cause negative influence on the safety characteristics of the blade in terms of flutter.

To obtain high-quality results at this stage of research you can solve this problem in the quasi-static approximation, that is, providing that the velocity change frequency \( V_t(t) \) is much lower than \( \omega_0 \) and \( \omega_{\phi} \), and acquiring constant \( V_t \) value for any moment of time. Maybe it should take the moment of time at which the \( V_t \) value is maximum given other parameters of the system have fixed values. In this case, if \( V_t = \text{const} \), system solution is:

\[
y_0 = y_{\text{forced}} + y_{\text{disturb}}, \quad \phi_0 = \phi_{\text{forced}} + \phi_{\text{disturb}},
\]
(10)

where \( y_{\text{forced}} \) and \( \phi_{\text{forced}} \) are partial solutions which describe forced constant oscillations of profile under the influence of external aerodynamic forces defined by terms given:

\[
\rho b V_t^2 \frac{\partial C_y}{\partial \alpha} \alpha(t),
\]

\[
(r_b - x_c) \rho b V_t^2 \frac{\partial C_y}{\partial \alpha} \phi_0(t).
\]
(11)

We are interested in disturbed motions of \( y_{\text{disturb}} \) and \( \phi_{\text{disturb}} \). They are defined as general solution of system of homogeneous equations which can be obtained from (9) by discarding explicitly time-dependent terms (11):

\[
-\dot{y}_0 + \epsilon_y \dot{y}_0 + \omega_y^2 y_0 - x_m \ddot{\phi}_0 = 0,
\]

\[
-\rho b V_t^2 \frac{\partial C_y}{\partial \alpha} \phi_0 - \frac{\dot{y}_0}{V_t} = 0,
\]

\[
\ddot{\phi}_0 + \epsilon_{\phi} \ddot{\phi}_0 + \omega_{\phi}^2 \phi_0 - x_m \dot{y}_0 = 0,
\]

\[
-(r_b - x_c) \rho b V_t^2 \frac{\partial C_y}{\partial \alpha} \phi_0 - \frac{\dot{y}_0}{V_t} = 0.
\]
(12)

Solution of system (12) is:

\[
y_{\text{disturb}} = y^0 e^{\epsilon_x t}, \quad \phi_{\text{disturb}} = \phi^0 e^{\epsilon_{\phi} t},
\]
(13)

where \( y^0 \) and \( \phi^0 \) are constant. Substituting (13) into (12) a system of two algebraic equations can be obtained, which have non-trivial solutions only if the
determinant of the system for some values of $\lambda$ vanishes. After this determinant is calculated, the characteristic equation of the fourth degree with respect to $\lambda$ can be obtained [8]. Four roots of this equation in general are complex

$$\lambda_k = q_k + i\rho_k \quad k = 1, 2, 3, 4.$$  \hspace{1cm} (14)

The imaginary parts $\rho_k$ of the roots (14) define oscillation frequencies, while real $q_k$ define the nature of how oscillations change in time: when $q < 0$ oscillations are convergent; when $q > 0$ they are increasing. Thus, the flutter can be estimated by the sign of the real parts of the roots $\lambda_k$.

Preliminary calculations show that the roots of (14) depend strongly on the angular speed of the rotor. Having reached critical velocity $\omega = \omega_{cr}$ the blade flutter occurs. The main influence on $\omega_{cr}$ is given by the positions of gravity center of elastic axis and front aerodynamic center in the chord and the ratio of frequencies $\omega_\phi$ and $\omega_{ho}$.

Relative position of these centers chosen in the wind turbine design is providing reliable flutter allowance. However, a comprehensive analysis of the problem based on the output nonlinear motion equations taking into account all possible factors which has been neglected to simplify the process model, requires further thorough research.

Another source of forced oscillations of the rotor blades can be aerodynamic forces pulsations caused by periodical breakdown of Karman’s vortices on the profile from the blade profile if it is installed at a large angle attack to the oncoming wind flow. Typically, these forces are determined as harmonic functions of time and breakdown vortices frequency, multiplied by some empirical coefficients.

It is known however that the breakdown vortices frequency as well as values of mentioned coefficients depend on the kinematic parameters of streamlined construction oscillations. The neglect of this fact makes it impossible to predict such an important phenomenon as synchronization of breakdown frequency with the oscillation frequency of the blade.

There can be proposed an analytical model of aerodynamic loads caused by breakdown of Karman’s vortices that can sufficiently simulate the behavior of aeroelastic constructions in a wide range of flow rates, amplitudes and frequencies of oscillations. In this model induced by breakdown vortices circulation around the profile is being described by potentially self-oscillatory equation:

$$\dot{\theta} + \mu \left[ \frac{F(\theta) - F_{cr}}{\varepsilon_1 + F_{cr}} \right] \dot{\theta} + \omega^2(F_{cr}) \theta = \varepsilon_2 \frac{b}{V} \alpha.$$  \hspace{1cm} (15)

Here $F(\theta)$ is an empirical function which describes the dependence of parameter $F$ flow or construction from breakdown vortices frequency from stationary profile, expressed by $\theta$; $\omega(F_{cr})$ is the breakdown vortices frequency as a function of the current value $F_{cr}$ of the parameter $F$. Particularly for wing profile the parameter $F$ corresponds to the full angle of attack $\alpha$, and the known experimental dependence $\omega(\alpha)$ of the wing can be approximated with sufficient accuracy by expression

$$\omega(\alpha) = \frac{2V_S}{b} \frac{\arctg\left(\frac{\alpha - \varphi_0}{a_1}\right)}{\pi S_{max}},$$  \hspace{1cm} (16)

where $a_1 = 30$, $S_{max} = 0.175$, $\varphi_0 = b^0$.

Hence, since $\theta(t)$ is a temporary form of circulation oscillations with amplitude $\theta_0 = \pi/2$, we find:

$$F(\theta) = \alpha(\theta) = \varepsilon_1 + a_1 \arctg\left(\frac{\theta}{\pi S_{max}}\right) - \varepsilon_1,$$  \hspace{1cm} (17)

$\alpha$ is the angle of attack, under which alternate vortices breakdown starts; $\varepsilon_1 = 18$, $\varepsilon_2 = 7.8$, $\mu = 0.1$ are empirical constants.

Breakdown vortices induce the angle of attack

$$\Delta \varphi_0 = \arctg \theta,$$  \hspace{1cm} (18)

which should be added to the full angle of attack (18); circulation is added to the dimensionless speed $b\alpha/V$. Thus, vortex formation parameters are also included in expressions $C^v_\alpha(\alpha, \dot{\alpha})$, $C^v_\beta(\alpha, \dot{\alpha})$ as arguments for non-stationary coefficients.

It is easy to see that the addition of equation (15) to system (6) makes it possible to investigate the behavior of the blades in all modes of movement, including the case of fixed rotor.

Last mentioned mode also requires investigation because of rotor stop during hurricane winds when vortices breakdown is the only source of non-stationary aerodynamic loads (except turbulent fluctuations of wind flow).

V. CONCLUSIONS

The wind turbines aeroelastic rotor vibrations calculation is made for Darrier rotor. It is proposed a new methodic of analysis which involves investigation of forced oscillations equations of horizontal blade section inside the space between cross-arms.
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Aviation Computer-Integrated Complexes Department, National Aviation University, Kyiv, Ukraine.
Education: Kyiv Polytechnic Institute, Kyiv, Ukraine (1973).
Publications: more than 500 papers.
E-mail: svm@nau.edu.ua

В. М. Синеглазов. Розрахунок аеропружних коливань роторів вітроенергетичних установ
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Ключові слова: аеропружні коливання; вітроенергетична установка; ротор; проектування.

Синеглазов Віктор Михайлович. Доктор технічних наук. Професор.
Кафедра авіаційних комп’ютерно-інтегрованих комплексів, Національний авіаційний університет, Київ, Україна.
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E-mail: svm@nau.edu.ua

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Синеглазов Віктор Михайлович. Доктор технічних наук. Професор.
Кафедра авіаційних комп’ютерно-інтегрованих комплексів, Національний авіаційний університет, Київ, Україна.
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E-mail: svm@nau.edu.ua