UDC 656.7.071: 656.7.052.002.5 (045)

M. Yu. Zaliskyi

RELIABILITY PARAMETERS ESTIMATION IN CASE OF AVIATION RADIO ELECTRONIC DEVICES TECHNICAL STATE DETERIORATION

Aviation radio electronic complexes department, National Aviation University, Kyiv, Ukraine
E-mail: maximus2812@ukr.net

***Abstract*—*The article describes procedure of statistical data processing in the operation system of aviation radio electronic devices in case of their technical state deterioration. Analysis of literature showed that there is a great variety of technical state deterioration models, among which the most commonly used is linear model. Therefore, reliability parameters estimates on the basis of the maximum likelihood method were obtained for linear model of technical state deterioration. Analysis of estimation procedure confirmed the mathematical correlations correctness.*

Index Terms—Aviation radio electronic devices; operation system; statistical data processing; estimation procedure.

I. INTRODUCTION

Aviation radio electronic devices (ARED) in civil aviation are used to provide flights in part of communication, navigation and surveillance, as well as to inspect passengers and baggage in the aviation security departments, etc. Reliable operation of these devices determines the efficiency of airlines.

The processes of ARED intended use, maintenance and repair, reliability parameters control and monitoring, implementing corrective and preventive actions are carried out in the corresponding operation systems (OS). The main elements of the ARED OS include these aviation radio electronic devices, current laws and regulations, support resources, and also procedures of data processing, decision-making based on technical state evaluation, etc. ARED state during the operation can change as a result of damages, failures, due to degradation processes, unstable power supplies, and other internal and external factors. This leads to the instability of air navigation services process fulfillment, changes its efficiency indexes, including the risks of air navigation services and risks of flight performing.

II. ANALYSIS OF PUBLICATIONS

Analysis of publications [1], [2] shows that today significant attention is paid to evaluating the reliability, operation systems design and implementation, determining relationship between maintenance and risks, etc. In operation practice, technical state of operation objects can deteriorate [3]. There are many models of technical state deterioration, but they are all associated with increased failure rate.

Problems of analysis of ARED technical state deterioration are typical problems of change-point detection and estimation [4], [5]. As for radio engineering application these problems considered in [6].

III. PROBLEM STATEMENT

The analysis of publications in the field of change-point detection and estimation showed that for ARED operation problems these questions are not enough discussed. Moreover, there is insufficiency of relatively simple engineering evaluation procedures.

The problems of reliability parameters estimation in the case of hopping model of ARED technical state deterioration were discussed in [7]. As known, it is the simplest model. But according to analysis [1], [3] linear model of technical state deterioration is most widespread in practice.

Therefore, this paper considers actual scientific and technical problem – synthesis and analysis of procedures for reliability parameters estimation in the case of a linear model of technical state deterioration.

IV. ESTIMATION PROCEDURE

It is known that for ARED there are three main stages of its operation: burn-in region (failure rate decreases), useful life region (with constant failure rate), and wear-out region (failure rate increases). This is a known concept used to represent failure changes of ARED because failure rate of such devices is a function of time [1]. Intersection between second and third stages is the beginning of technical state deterioration process.

Hazard rate function on the second and third stages of operation for linear model is described by expression

$$\lambda(x)=\begin{cases}\lambda, & \text{if } x<k;\\ \lambda+2ax, & \text{if } x\ge k,\end{cases}$$

where λ is constant failure rate ($\lambda>0$), a is coefficient of technical state deterioration ($a>0$), k is the moment of technical state deterioration beginning (i.e. the number of recorded failures before the beginning of technical state deterioration).

Let's consider the problem of λ and a estimation.

The initial data for the estimation of reliability parameters is time to failure x_i, where i is number of recorded failure. Let's assume that the total number of failures is $n > k$.

According to basic equations of reliability [1] the relationship between failure rate and the probability of failure-free operation (reliability function) $R(x)$ is following:

$$R(x) = e^{-\int_0^x \lambda(x)dx}..$$

Substituting in this formula the expression for hazard rate function $\lambda(x)$, we'll get reliability function

$$R(x) = \begin{cases} e^{-\lambda x}, & \text{if } x < k; \\ e^{-\lambda x - ax^2}, & \text{if } x \geq k. \end{cases}$$

Then we can find the expression for the probability density function (PDF) of times between failures

$$f(x) = -\frac{dR(x)}{dx} = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x < k; \\ (\lambda + 2ax)e^{-\lambda x - ax^2}, & \text{if } x \geq k. \end{cases}$$

Let us consider the problem of λ estimation in case of known value of coefficient of technical state deterioration a ($a > 0$). To solve this problem, we can use the maximum likelihood method, method of moments, quantile method, least squares method, etc. In this paper, we use the maximum likelihood method.

Likelihood function for investigated options of technical state deterioration can be written in the following form:

$$\Lambda(\lambda, a, k) = \prod_{i=1}^{k} \lambda e^{-\lambda x_i} \prod_{i=k+1}^{n} (\lambda + 2ax_i) e^{-\lambda x_i - ax_i^2}$$

$$= \lambda^k \left(\prod_{i=k+1}^{n} (\lambda + 2ax_i) \right) e^{-\lambda \sum_{i=1}^{n} x_i - a \sum_{i=k+1}^{n} x_i^2}.$$

To find the unknown parameter λ estimate, let's solve the equation

$$\frac{d\Lambda(\lambda, a, k)}{d\lambda} = 0.$$

In this case

$$\frac{d\Lambda(\lambda, a, k)}{d\lambda} = k\lambda^{k-1} \left(\prod_{i=k+1}^{n} (\lambda + 2ax_i) \right) e^{-\lambda \sum_{i=1}^{n} x_i - a \sum_{i=k+1}^{n} x_i^2} - \lambda^k \left(\prod_{i=k+1}^{n} (\lambda + 2ax_i) \right) e^{-\lambda \sum_{i=1}^{n} x_i - a \sum_{i=k+1}^{n} x_i^2} \sum_{i=1}^{n} x_i + \lambda^k r e^{-\lambda \sum_{i=1}^{n} x_i - a \sum_{i=k+1}^{n} x_i^2}.$$

where

$$r = \sum_{j=0}^{n-k-1} \left(\prod_{i=k+1}^{k+1+j} (\lambda + 2ax_i) \prod_{i=k+3+j}^{n-k-1-j} (\lambda + 2ax_i) \right)$$

$$+ \prod_{i=k+2}^{n} (\lambda + 2ax_i) + \prod_{i=k+1}^{n-1} (\lambda + 2ax_i).$$

After simplifications, we obtain equation

$$\left(k - \lambda \sum_{i=1}^{n} x_i \right) \prod_{i=k+1}^{n} (\lambda + 2ax_i) + \lambda r = 0.$$

This equation can be rewritten as

$$k - \lambda \sum_{i=1}^{n} x_i + \sum_{i=k+1}^{n} \frac{\lambda}{\lambda + 2ax_i} = 0.$$

Obtained equation can be solved for the two limiting cases: 1) when $\lambda >> 2ax_i$; 2) and when $\lambda << 2ax_i$.

In the first case, we obtain the estimate

$$\lambda = \frac{n}{\sum_{i=1}^{n} x_i}. \quad (1)$$

For this estimate, we can quite easily find PDF, mathematical expectation and variance:

$$f(x) = \frac{(n\lambda)^n}{x^{n+1}(n-1)!} e^{-\frac{\lambda n}{x}};$$

$$m_1(x) = \frac{n\lambda}{n-1};$$

$$\mu_2(x) = \frac{(n\lambda)^2}{(n-1)^2 (n-2)}.$$

In the second case, we obtain the estimate

$$\lambda = \frac{k}{\sum_{i=1}^{n} x_i - \frac{1}{2a} \sum_{i=k+1}^{n} \frac{1}{x_i}}. \quad (2)$$

Let's consider the problem of a estimation in case of known value of failure rate λ.

For this purpose, it is necessary to solve the equation

$$\frac{d\Lambda(\lambda, a, k)}{da} = 0.$$

In this case

$$\frac{d\Lambda(\lambda,a,k)}{da}=\lambda^k r_1 e^{-\lambda\sum_{i=1}^{n}x_i-a\sum_{i=k+1}^{n}x_i^2}$$

$$-\lambda^k\left(\prod_{i=k+1}^{n}(\lambda+2ax_i)\right)e^{-\lambda\sum_{i=1}^{n}x_i-a\sum_{i=k+1}^{n}x_i^2}\sum_{i=k+1}^{n}x_i^2,$$

where

$$r_1=\sum_{j=0}^{n-k-1}\left(2x_{k+2+j}\prod_{i=k+1}^{k+1+j}(\lambda+2ax_i)\prod_{i=k+3+j}^{n-k-1-j}(\lambda+2ax_i)\right)$$

$$+2x_{k+1}\prod_{i=k+2}^{n}(\lambda+2ax_i)+2x_n\prod_{i=k+1}^{n-1}(\lambda+2ax_i).$$

After simplifications, we obtain equation

$$\left(-\sum_{i=k+1}^{n}x_i^2\right)\prod_{i=k+1}^{n}(\lambda+2ax_i)+r_1=0.$$

This equation can be rewritten as

$$\sum_{i=k+1}^{n}x_i^2-\sum_{i=k+1}^{n}\frac{2x_i}{\lambda+2ax_i}=0.$$

Quite simple solution of this equation can be obtained in case of $\lambda \ll 2ax_i$, then

$$\frac{2x_i}{\lambda+2ax_i}\approx\frac{1}{a}.$$

So a parameter estimate

$$a=\frac{n-k}{\sum_{i=k+1}^{n}x_i^2}. \qquad (3)$$

Another variant of estimation can be obtained from the fraction decomposition in a Taylor series (if we take two first terms of this series)

$$\frac{2x_i}{\lambda+2ax_i}\approx\frac{1}{a}-\frac{\lambda}{2a^2x_i}.$$

Then

$$\sum_{i=k+1}^{n}x_i^2-\sum_{i=k+1}^{n}\left(\frac{1}{a}-\frac{\lambda}{2a^2x_i}\right)=0.$$

Hence

$$a^2\sum_{i=k+1}^{n}x_i^2-(n-k)a+\frac{\lambda}{2}\sum_{i=k+1}^{n}x_i^{-1}=0.$$

Parameter a estimate looks like

$$a=\frac{n-k+\sqrt{(n-k)^2-2\lambda\sum_{i=k+1}^{n}x_i^2\sum_{i=k+1}^{n}x_i^{-1}}}{2\sum_{i=k+1}^{n}x_i^2}. \qquad (4)$$

The simulation results showed that estimate (4) is also suitable only for cases when $\lambda \ll 2ax_i$.

Let us consider simultaneous a and λ estimation procedure. For this purpose, it is necessary to solve a system of equations

$$\begin{cases}\dfrac{d\Lambda(\lambda,a,k)}{d\lambda}=0,\\ \dfrac{d\Lambda(\lambda,a,k)}{da}=0.\end{cases}$$

Thus, we get

$$\begin{cases}k-\lambda\sum_{i=1}^{n}x_i+\sum_{i=k+1}^{n}\dfrac{\lambda}{\lambda+2ax_i}=0;\\ \sum_{i=k+1}^{n}x_i^2-\sum_{i=k+1}^{n}\dfrac{2x_i}{\lambda+2ax_i}=0.\end{cases}$$

If from the first equation we subtract the second equation multiplied by a, we'll get

$$k-\lambda\sum_{i=1}^{n}x_i-a\sum_{i=k+1}^{n}x_i^2+\sum_{i=k+1}^{n}\frac{\lambda+2ax_i}{\lambda+2ax_i}=0.$$

Then

$$k-\lambda\sum_{i=1}^{n}x_i-a\sum_{i=k+1}^{n}x_i^2+n-k=0,$$

or

$$n-\lambda\sum_{i=1}^{n}x_i-a\sum_{i=k+1}^{n}x_i^2=0,$$

Solving this equation with respect to a and with respect to λ, we obtain

$$\lambda=\frac{n-a\sum_{i=k+1}^{n}x_i^2}{\sum_{i=1}^{n}x_i}. \qquad (5)$$

$$a=\frac{n-\lambda\sum_{i=1}^{n}x_i}{\sum_{i=k+1}^{n}x_i^2}. \qquad (6)$$

Equations (5) and (6) can be used as estimates of these parameters, if one of them is known in advance.

Let's perform analysis of the failure rate estimates (1), (2) and (5), and also coefficient of technical state deterioration (3), (4) and (6) by modeling.

The following values were chosen as initial data for modeling: failure rate $\lambda = 10^{-3}$, sample size $n = 1000$, the beginning of technical state deterioration $k = 500$, the number of procedures simulation repetitions $N = 100$.

The simulation results in the form of λ estimates PDF according to formulas (1) and (5), if $a = 10^{-8}$, are shown in Figs 1 and 2, respectively.

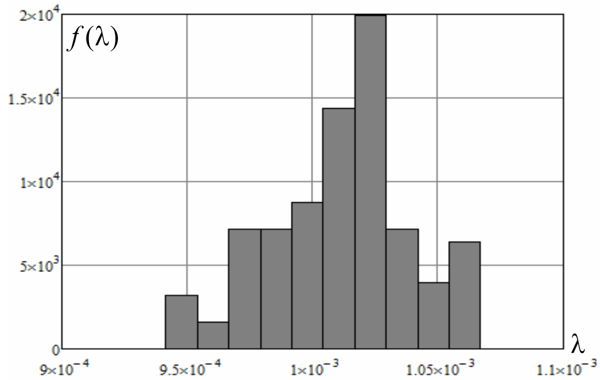

Fig. 1. PDF of λ estimates according to (1)

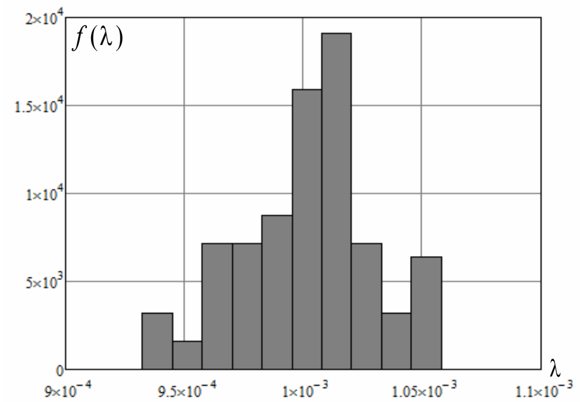

Fig. 2. PDF of λ estimates according to (5)

For procedure (1) mathematical expectation and variance of estimate: $m_1(\lambda) = 1{,}012 \cdot 10^{-3}$, $\mu_2(\lambda) = 7{,}729 \cdot 10^{-10}$; and for procedure (5): $m_1(\lambda) = 1{,}002 \cdot 10^{-3}$, $\mu_2(\lambda) = 7{,}828 \cdot 10^{-10}$. As can be seen if $\lambda >> 2ax_i$, both procedures give approximately the same results. Procedure (2) at the same time gives unsatisfactory results; it can be used only, if $\lambda << 2ax_i$. According to the simulation results in a wide range of initial data, we can conclude that the best among considered procedures is procedure (5). This procedure usage is possible in the case of an arbitrary relationship between a and λ.

The simulation result in the form of a estimates PDF according to formulas (6), if $a = 10^{-6}$, is shown in Fig. 3.

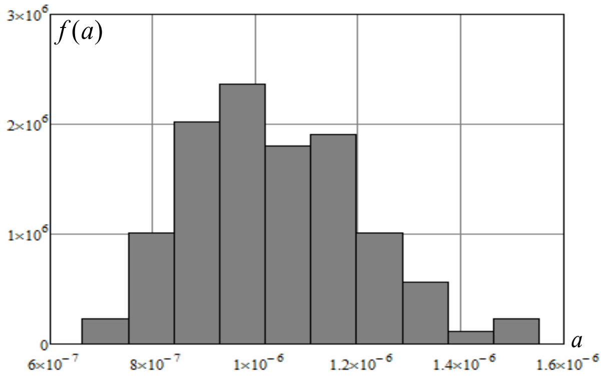

Fig. 3. PDF of a estimates according to (6)

For procedure (6) mathematical expectation and variance of estimate: $m_1(a) = 1{,}032 \cdot 10^{-6}$ and $\mu_2(a) = 2{,}935 \cdot 10^{-14}$. Analysis of the modeling parameter a estimation procedure showed the possibility of using the algorithm (6) in a wide range of initial data. Obtained by the expressions (3) and (4) estimates give acceptable results, only if $\lambda << 2ax_i$.

V. CONCLUSION

Actual problems of procedures synthesis for data processing in the operation systems of aviation radio electronic devices are considered in the article. In the case of the linear model of technical state deterioration, failure rate and coefficient of technical state deterioration estimates were obtained using the maximum likelihood method. Estimation procedures analysis by mathematical modeling confirmed the possibility of these procedures usage.

Obtained results can be used in the design, implementation and modernization of operation systems of aviation radio electronic devices.

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Received July 16, 2015

Maksym Zaliskyi. PhD.
Aviation radio electronic complexes department, National Aviation University, Kyiv, Ukraine.
Education: National Aviation University, Kyiv, Ukraine (2007).
Research area: maintenance, data processing, radio engineering.
Publications: 51.
E-mail: maximus2812@ukr.net

М. Ю. Заліський. Оцінювання параметрів надійності під час погіршення технічного стану авіаційних радіоелектронних засобів

Розглянуто процедуру статистичної обробки даних в системах експлуатації авіаційних радіоелектронних засобів у випадку погіршення їх технічного стану. Аналіз літературних джерел показав, що існує велике різноманіття моделей погіршення технічного стану, серед яких найбільш уживаною є лінійна модель. Тому саме для лінійної моделі погіршення технічного стану були отримані оцінки показників надійності на основі використання методу максимальної правдоподібності. Проведений аналіз процедури оцінювання підтвердив правильність математичних співвідношень.

Ключові слова: авіаційні радіоелектронні засоби; система експлуатації; статистична обробка даних, оцінювання.

Заліський Максим Юрійович. Кандидат технічних наук.
Кафедра авіаційних радіоелектронних комплексів, Національний авіаційний університет, Київ, Україна.
Освіта: Національний авіаційний університет, Київ, Україна (2007).
Напрям наукової діяльності: системи експлуатації, обробка даних, радіоелектронне обладнання.
Кількість публікацій: 51.
E-mail: maximus2812@ukr.net

М. Ю. Залисский. Оценивание параметров надёжности в случае ухудшения технического состояния авиационных радиоэлектронных средств

Рассмотрена процедура статистической обработки данных в системах эксплуатации авиационных радиоэлектронных средств при условии ухудшения их технического состояния. Анализ литературных источников показал, что существует большое разнообразие моделей ухудшения технического состояния, среди которых наиболее используемой является линейная модель. Потому именно для линейной модели ухудшения технического состояния были получены оценки показателей надёжности с использованием метода максимального правдоподобия. Проведенный анализ процедуры оценивания подтвердил правильность математических соотношений.

Ключевые слова: авиационные радиоэлектронные средства, система эксплуатации, статистическая обработка данных, оценивание.

Залисский Максим Юрьевич. Кандидат технических наук.
Кафедра авиационных радиоэлектронных комплексов, Национальный авиационный университет, Киев, Украина.
Образование: Национальный авиационный университет, Киев, Украина (2007).
Направление научной деятельности: системы эксплуатации, обработка данных, радиоэлектронное оборудование.
Количество публикаций: 51.
E-mail: maximus2812@ukr.net