MATHEMATICAL MODELING OF PROCESSES AND SYSTEMS

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A PROCEDURE OF BUILDING A COMPUTER MODEL OF AN OPTOELECTRONIC MEASURING SYSTEM

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The work deals with modeling of an optoelectronic vibration measuring system at the level of a functional diagram by means of MATLAB tools.

Keywords: system modeling, vibration measurement, MATLAB.

Introduction. Statement of the problem. Modeling systems at the level of a functional diagram is widely used in the design practice [1]. Currently, the literature suggests making such modeling in the MATLAB environment using the SIMULINK application [2]. This application is particularly helpful in analyzing automation systems. However, in modeling radar systems, which involves complicated processing of complex signals, the use of SIMULINK results in unnecessary complication of the model, since it does not have a program library of complex signals. In this case, programming in the MATLAB core allows a more flexible use of the huge potential of its environment for building more adequate models of radar systems.

The subject of this work is modeling of an optoelectronic laser system of vibration measurement at the level of a functional diagram by means of MATLAB tools. Such systems are used both in industrial processes and as directional microphones. Of course, the proposed approach to modeling can also be applied for analyzing other radar systems.

For remote laser measurement of low-frequency vibration characteristics a variety of measuring methods are used. In this paper we consider the phase method of measurement.

The block diagram of the measuring device is shown in fig. 1.

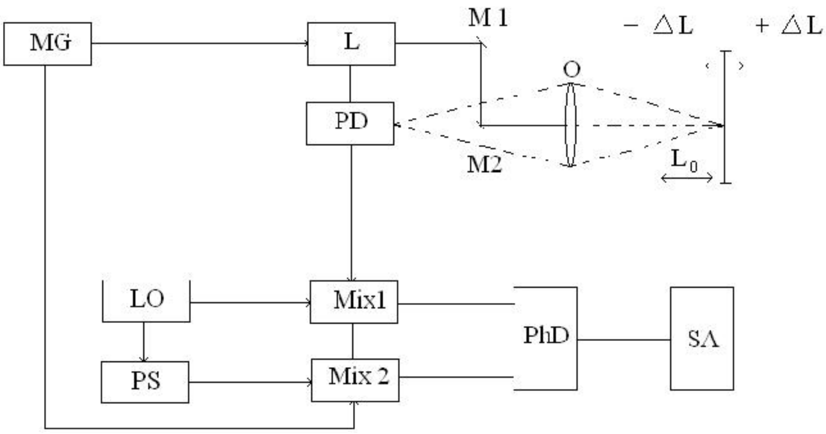

Fig. 1. The block diagram of a vibration meter using the phase method: MG – modulation generator; L – laser; PD – photodetector; LO – local oscillator; PS – 90º phase shifter; Mix1 – the first mixer; Mix 2 – the second mixer; PhD – phase detector; O – optical system; M1 and M2 – mirrors; SA – spectrum analyzer

In modeling a laser optoelectronic vibration measuring system, its optical and electronic channels are analyzed separately.

The mathematical model of the optical channel. The measuring signal is generated as follows.

The laser beam L reflected from the mirrors M1 and M2 is focused by the optical system O and covers the distance L_0 to the glass (L_0 may be up to 100 m). The laser beam reflected from the glass is focused by the optical system on the photodetector PD. The output voltage U_{PD} of the photodetector depends on the laser power P_0, the optical path loss factor k_{OP} and the sensitivity of the photodetector S_{PD}.

$$U_{PD} = P_0 \cdot k_{OP} \cdot S_{PD}. \quad (1)$$

The laser radiation power is modulated by the modulating voltage generator MG

$$U_M = U_{M0} \sin(2\pi f_M t) \quad (2)$$

and has a constant component $P^{(1)}$ and a variable component

$$P^{(2)} = P \sin(2\pi f_M t). \quad (3)$$

As a result, the useful output voltage of the photodetector has a variable component

$$U_{PD} = Pk_{OP}S_{PD} \sin(2\pi f_M t) = U_{PD.O} \sin(2\pi f_M t). \quad (4)$$

Shifting the reflector plane by ΔL leads to a phase change of the output signal of the photodetector

$$\Delta\varphi = 4\pi \frac{\Delta L}{\lambda}, \quad (5)$$

where $\lambda = \frac{c}{f_M}$.

If the reflector plane vibrates with a frequency f_V

$$\Delta L = \Delta L_0 \sin(2\pi f_V t), \quad (6)$$

the phase of the photodetector output signal undergoes the modulation:

$$U_{PD} = U_{PD,0} \sin\left[2\pi f_M t + \frac{4\pi\Delta L_0}{\lambda}\sin(2\pi f_V t)\right] \quad (7)$$

This phase-modulated signal is generally hidden in a noise (both additive and multiplicative). In this paper we suggest the following description of the noise:

$$U_{PD}^{(INT)} = U_{PD.0}[1 - k_1 + k_2 U_{U,D}] \sin[2\pi f_M t + \frac{4\pi\Delta L_0}{\lambda}\sin(2\pi f_V t) + k_3 U_{G.D.}], \quad (8)$$

where k_1 – coefficient defining the overall increase in optical channel losses $0 \le k_1 \le 1$; k_2 – coefficient defining the amplitude of fluctuations $k_2 \le k_1$; k_3 – coefficient defining the dispersion of additive noise; $U_{U,D}$ – a random signal whose probability density is uniformly distributed in the range [0…1]; $U_{G.D.}$ – a random signal whose Gaussian probability density distribution and variance are 1.

This high-frequency complex signal electronically processed, we obtain a low-frequency information component whose characteristics are measured by a spectrum analyzer.
For this purpose the phase-modulated signal is decreasingly converted in frequency by means of the mixer Mix 1 and LO, then fed to the phase detector PhD. The phase detector reference signal of the same intermediate frequency is formed by mixing the MG and LO signals by the mixer Mix 2. The phase required for PD operation is set by the phase shifter (PS) at 90º. The phase detector demodulates the signal, thus extracting its information component.

The functional diagram of the measuring system used for the modeling is shown in fig. 2. This model has the following features:

1. The photodetector is modeled by a generator of the photodetector output signal (unit 1) according to (8) and is a connecting link between the model of the optical cannel and the model of the electronic cannel.

2. The phase shifter is modeled by a local oscillator LO-2 (unit 4) generating a harmonic signal shifted by 90 degrees relative to the local oscillator LO-1 signal (unit 3).

3. The mixer 1 is modeled by a series connection of a signal multiplier (unit 5) and a band-pass filter (unit 6).

4. The mixer 2 is modeled by a series connection of a signal multiplier (unit 7) and a band-pass filter (unit 8).

5. The phase detector is modeled by a series connection of a signal multiplier (unit 9) and a low-pass filter (unit 10).

6. The spectrum analyzer (unit 11) is modeled by means of Fast Fourier transform (FFT) of MATLAB (fft).

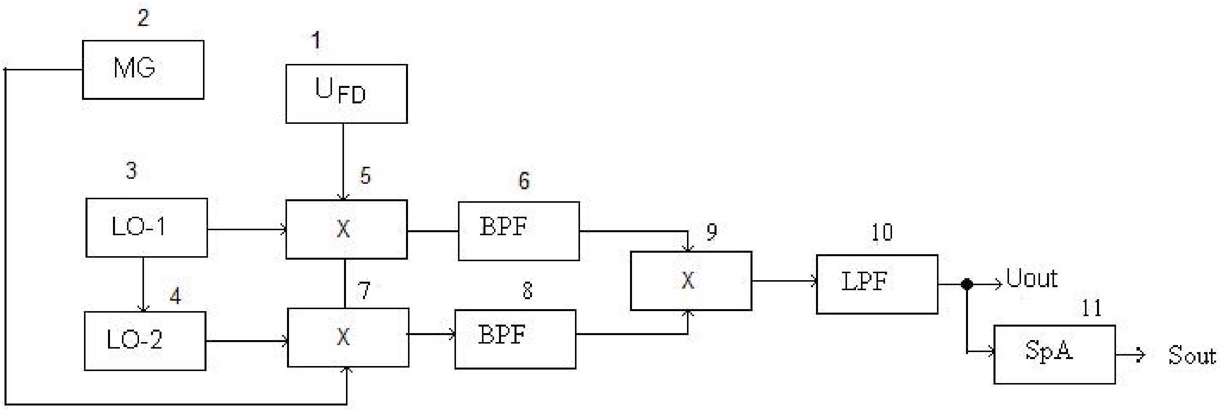

Fig. 2. The functional diagram of a vibration meter using the phase method

The mathematical model of the electronic cannel of the measuring system is a mathematical description of signal transformation according to the functional diagram (fig. 2).

The local oscillator LO-1 generates a voltage U_g

$$U_g = U_{go}\sin(2\pi f_g t), \tag{9}$$

where $f_g = f_m + f_{\text{int}}$, f_{int} – intermediate frequency.

Voltages U_g и U_{PD} arrive at the signal multiplier (unit 5).

The output signal of the multiplier is

$$U_{MIX1}^{MUL} = aU_{g.O}U_{PhD.O}\sin(2\pi f_g t)\sin[2\pi f_M t + \frac{4\pi\Delta L_0}{\lambda}\sin(2\pi f_V t)] =$$

$$= \frac{1}{2}aU_{gO}U_{PD.O}\cos[2\pi(f_g - f_M)]t - \frac{4\pi\Delta L_0}{\lambda}\sin(2\pi f_V t) -$$

$$\frac{1}{2}aU_{gO}U_{PD.O}\cos[2\pi(f_g + f_M)t + \frac{4\pi\Delta L_0}{\lambda}\sin(2\pi f_V t)], \tag{10}$$

where a – a hardware constant whose dimension is $[V^{-1}]$. After filtration made by the bandpass filter (unit 6) tuned to the frequency $f_{\text{int}} = f_g + f_M$, at the output of the mixer Mix1 (at the first input of the phase detector) we get:

$$U_{FD.1} = \frac{1}{2}aU_{gO}U_{PD.O}\cos[2\pi f_{\text{int}}t - \frac{4\pi\Delta L_0}{\lambda}\sin(2\pi f_V t)]. \tag{11}$$

The local oscillator LO-2 generates a voltage U_{g2}

$$U_{g2} = U_{gO}\cos(2\pi f_g t). \tag{12}$$

Similarly to mixer Mix1 operation, at the output of the mixer Mix2 (at the second input of the phase detector) we get:

$$U_{FD.2} = -\frac{1}{2} a U_{gO} U_{MO} \sin[2\pi f_{\text{int}} t]. \tag{13}$$

The signals $U_{FD.1}$ and $U_{FD.2}$ arrive at the phase detector's multiplier of signals (unit 9). The output signal of phase detector's multiplier is:

$$\begin{aligned} U_{PD}^{MUL} &= -\frac{1}{4} a^2 U_{gO}{}^2 U_{MO} U_{PD.O} \sin(2\pi f_{\text{int}} t)\cos\left[2\pi f_{\text{int}} t - \frac{4\pi\Delta L_0}{\lambda}\sin(2\pi f_V t)\right] = \\ &= -\frac{1}{8} a^3 U_{gO} U_{MO} U_{PD.O} \sin\left[\frac{4\pi\Delta L_0}{\lambda}\sin(2\pi f_V t)\right] - \\ &-\frac{1}{8} a^3 U_{gO} U_{MO} U_{PD.O} \sin\left[4\pi f_{\text{int}} t - \frac{4\pi\Delta L_0}{\lambda}\sin(2\pi f_V t)\right]. \end{aligned} \tag{14}$$

After the output signal of the phase detector's multiplier has been filtered we get the output signal of the phase detector:

$$U_{OUT} = -\frac{1}{8} a^3 U_{g.O} U_{M.O} U_{PD.O} \sin\left[\frac{4\pi\Delta L_0}{\lambda}\sin(2\pi f_V t)\right]. \tag{15}$$

Since $\frac{4\pi\Delta L_0}{\lambda} \ll 1$,

$$U_{OUT} = -\frac{1}{8} a^3 U^2{}_{g.O} U_{M.O} U_{PD.O} \frac{4\pi\Delta L_0}{\lambda}\sin(2\pi f_V t) = -U_{OUT.0}\sin(2\pi f_V t). \tag{16}$$

The amplitude and frequency of the output signal of the phase detector are analyzed by the spectrum analyzer.

Filtration simulation. There are many procedures in MATLAB that simulate filtration. In this work filtration is performed in the frequency domain, and the spectrum of the filtered signal is

$$\dot{S}_f = \dot{S}_{inp} \cdot \dot{K}(\omega) \tag{17}$$

where $\dot{S}_{inp}$ – the signal spectrum at the filter input; $\dot{S}_f$ – the signal spectrum at the filter output; $\dot{K}(\omega)$ – frequency-related band-pass filter gain.

In the processing algorithm a filter with no phase shift is used:

$$\dot{K}(\omega) = 1 \text{ in the pass band, } \dot{K}(\omega) = 0 \text{ in the stop band.} \tag{18}$$

The spectrum of the input signal is calculated by the FFT algorithm. The time function of the filter output signal is calculated by the Inverse Fast Fourier transform (IFFT) algorithm.

The functional diagram of a filter is shown in fig. 3.

In the case of a BPF the bandwidth of the filter's gain is taken in the vicinity of the intermediate frequency. In the case of a LPF the bandwidth of the filter's gain is taken depending on the vibration frequency.

This functional model of an optoelectronic laser vibration measurement system, described by expressions (1) – (18), is implemented in a computer program within the MATLAB system in the form of Script-File. It fully describes the signal processing and can be used to evaluate system operation under noise conditions.

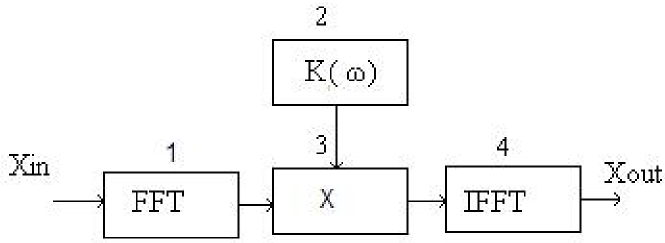

Fig. 3. The functional diagram of a filter

Model analysis possibilities within the MATLAB environment have been tested under the following conditions:

1) $f_M = 25$ MHz, $f_{\text{int}} = 80$ kHz, $f_V = 8$ kHz and 7,9 kHz;

2) $U_{PD.0}$ =1, oscillation amplitudes of the generators are 10 times greater than $U_{PD.0}$;.

3) Vibration amplitudes ΔL_0 =1mm….1 μm, or $\frac{4\pi\Delta L_0}{\lambda} = 10^{-3}\ldots10^{-6}$;

4) Time interval in which simulation is carried out T = 0,01 s.

The frequencies are taken according to [3]. Testing was carried out in the absence of noise, and it showed that the possibilities of computer analysis in MATLAB are limited by:

– rounding errors;
– program memory overflow errors;
– oscillation spectrum calculation errors.

Rounding errors arise due to the fact that in modeling a phase-modulated signal according to (8) two values are added the difference between which constitutes many orders of magnitude. These are a great phase of the carrier oscillation $2\pi f_M t$ and a small increment to it $\frac{4\pi\Delta L_0}{\lambda}\sin(2\pi f_B t)$. Since only a limited number of significant digits of the addition result remains in the computer, machine rounding brings about an error in describing the modulating oscillation. This error increases with decreasing the modulation factor $m = \frac{4\pi\Delta L_0}{\lambda}$.

The calculated signal spectrum for different values of the modulation factor is shown in fig. 4 – 6.

An error in frequency measurement by a spectrum analyzer, based on signal processing by the FFT algorithm, occurs when the oscillation frequency is not multiple of the frequency discrete Δf = 100 Hz. The reasons for this lie in the FFT algorithm and are described in [2]. An example of an error in frequency determination is shown in fig. 7.

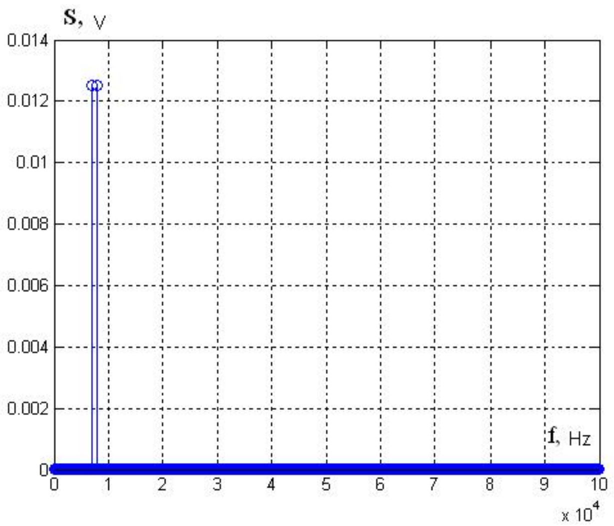

Fig. 4. The calculated spectrum of vibrations for m= 10^{-3}

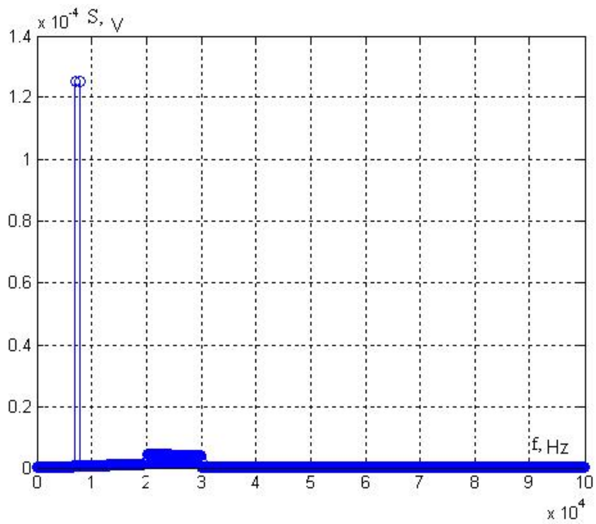

Fig. 5. The calculated spectrum of vibrations for m = 10^{-5}

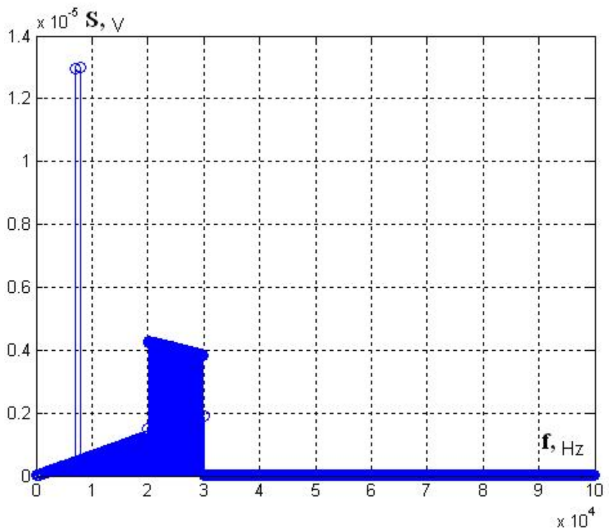

Fig. 6. The calculated spectrum of vibration for m= 10^{-6}.

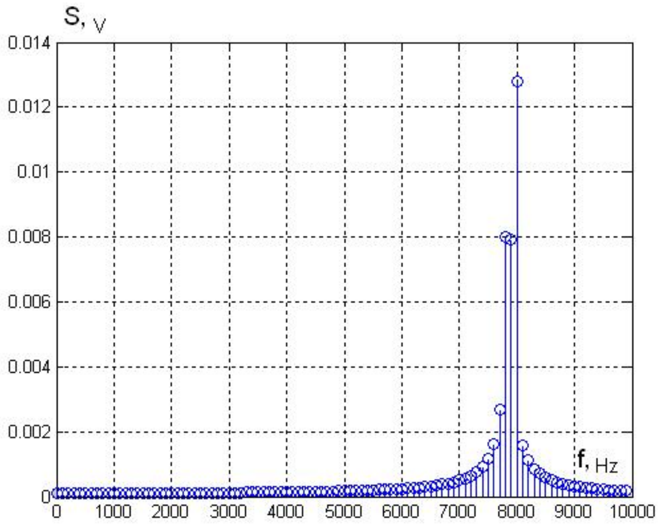

Fig. 7. An error in vibration frequency measurement for f_V = 8 kHz and 7,85 kHz.

A program memory overflow error occurs due to a large one-dimensional array, which stores the values of the phase-modulated signal. This size results from the fact that in the process of simulation 10 samples are taken for a period of the carrier oscillation, and the time interval in which the simulation is carried out is taken T = 0,01 to increase the frequency distinguishability by the spectrum analyzer built on signal processing by the FFT algorithm, the frequency discrete being Δf = 100 Hz.

With further increasing T to 0,015 s and decreasing Δf the program generates a memory overflow error message:

Error using ==> fft Out of memory.

The results of studying the noise immunity of the measurement system. Taking account of the above limitations, we used a computer model to assess the system performance under noise conditions. The noise values were specified by the coefficients k_1, k_2, k_3 in (8). We took the following parameters: band-pass filter bandwidth **П** = 50 ... 100 kHz, low-pass filter bandwidth **П** = 0 ... 30 kHz. Fig. 8 shows the calculated spectrum of the signal under multiplicative noise, fig. 9 – under additive noise.

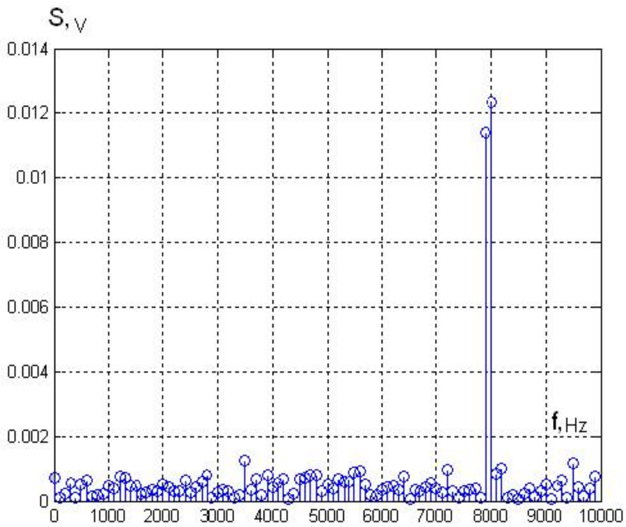

Fig. 8 The calculated vibration spectrum under multiplicative noise (m = 10^{-3}, k_1 = 0,1; k_2 = 10^{-1}, k_3 = 0)

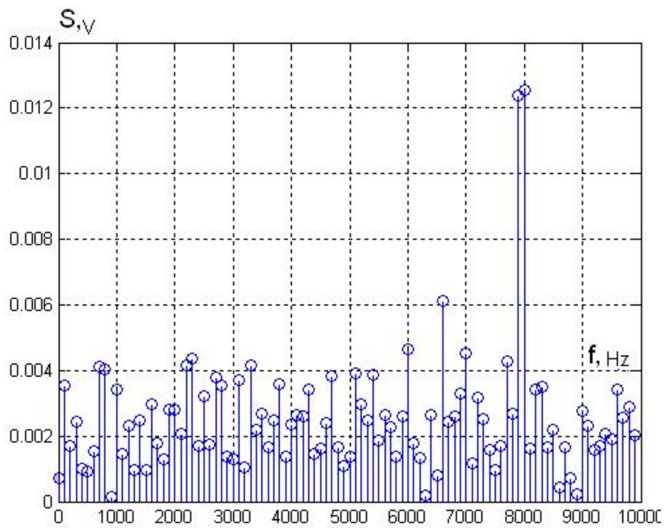

Fig. 9 The calculated vibration spectrum under additive noise (m = 10^{-3}, k_1= 0; k_2= 0, k_3 = 10^{-1})

As you can see, with such significant noise intolerable errors occur in measuring the parameters of real vibrations and manifest themselves in the wrong determination of oscillations. At the same time, the examples show considerable noise immunity of signal processing algorithms.

Conclusions. A procedure for constructing computer models of optoelectronic systems at the level of functional circuits has been developed.

A functional model of a laser vibration measurement system has been developed using the MatLAB tools.

Mathematical models of additive and multiplicative noise that interfere laser system operation have been suggested.

Possibilities of a program for analyzing the effect of additive and multiplicative noise on a laser vibration meter have been studied.

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Г. Є. Соколов

Методика побудови компьютерної моделі оптично-електронної вимірювальної системи

Проведено моделювання на рівні функціональної схеми оптично-електронної системи вимірювання вібрації засобами MATLAB.

Г. Е. Соколов

Методика построения компьютерной модели оптико-электронной измерительной системы

Проведено моделирование на уровне функциональной схемы оптико-электронной системы измерения вибраций средствами MATLAB.