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APPROXIMATE CALCULATION OF THE PROCESS CHARACTERISTICS
OF THE UAV LANDING ON ROPE

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Abstract—Analytical expressions, which connected of the unmanned aerial vehicle parameters and
characters of landing's rope, are obtained. In particular, the relationship between the necessary
length of rope and the value of its stretching during the landing of the unmanned aerial vehicle is
determined. Meanwhile, additional dampers for the rope are not considered. The mathematical model of
the deceleration process of unmanned aerial vehicle during its landing on rope is based on Hooke's law
and Newton's 2nd law. One of the main assumptions at the development of the mathematical model is that
of the braking of the unmanned aerial vehicle after its coupling with the rope occurs with constant
acceleration.

Index Terms—Unmanned aerial vehicle landing; rope stretching; Hooke's law; unmanned aerial vehicle
overload.

I. INTRODUCTION

The unmanned aerial vehicles, launched from the
hands or from the starting guide track installation
(catapult), must have defined means for landing.
Sometimes, the Unmanned Aerial Vehicle (UAV)
launched in this way are equipped with a parachute,
with the help of which they are down to earth.

In recent years, specially designed landing
methods for UAVs without chassis have been
developed and applied: landing on rope or in a
special elastic net [1] – [3]. As a result of application
of these methods is increasing economic and
technical efficiency of UAV by increasing its
payload due to the lack of chassis on board. In
addition, the UAV landing with the help of devices,
which are realized such methods, can be carried out
on area of limited dimensions in a given place.

Known landing device [1], [3] contain: a rope
stretched perpendicular to the trajectory of UAV on
a place of landing, a brake hook and a shock
absorber located on the UAV. The device [1] has
also first and second gates. Schematically two main
moments of UAV landing on this device is shown in
Fig. 1. In Figure 1a is shown moment, when
the kinetic energy of UAV is equal zero and in Fig. 1b
is shown final moment of UAV landing, when the UAV
is immovable. The height of the rope of first gate $H_1$
is greater than the height of second one $H_2$ on the
value $\Delta H = H_1 - H_2$, so that

$$\Delta H \geq \frac{gL_u^2}{(2V_{fV}^2)},$$

where $g$ is the acceleration of gravity; $L$ is the
distance between landing gates, which depends on
the rigidity of the brake rope and the characteristics
of the shock absorber; $V_{fV}$ is the UAV landing speed.

Fig. 1. The two moments of UAV landing:
1 is the hook; 2 is the elastic brake rope; 3 is the first gate;
4 are dampers; 5 is the second gate; 6 is the nonelastic
rope; 7 is the UAV
II. PROBLEM STATEMENT

The purpose of this work is development of calculation method of the UAV parameters landing on the elastic brake rope.

Assume that trajectory of the UAV before landing is in horizontal plane and at the UAV landing on the brake rope; its kinetic energy is parried by elastic ropes only.

Let the elastic rope between the supports of the brake gates A and B have length \( L \) before UAV landing and coupling of hook to rope is executed at point O, as it shown in Fig. 2.

![Diagram](image)

Fig. 2. To calculation of rope stretching during UAV landing: \( \Delta l \) is the transversal lengthening of rope; vector \( F \) is the force, which is imposed to point O by the UAV at landing; vector \( F_A, F_B \) are the forces, which lengthen out the rope from the left side and the right sides

III. PROBLEM SOLUTION

Assume that when the rope is hooked, an absolutely inelastic impact occurs, i.e. the speeds of the UAV and of the rope at the point of coupling become equal. Then a uniformly decelerated motion of the UAV with the rope on distance \( \Delta l \) during time \( t \) occurs. As a result, the UAV speed is reduced to zero. Conditionally the complete solution of the problem can be divided into two parts:

1) determination of the kinematic parameters of the UAV motion while landing on the rope;
2) determination of the dynamic parameters of the UAV motion.

To calculate the kinematic parameters of the UAV motion, we write two equations with two unknowns:

\[
\begin{align*}
V_{FV}(t) &= at, \\
\Delta l &= \frac{at^2}{2},
\end{align*}
\]

where \( a \) is the acceleration of breaking action of UAVs at its coupling with a rope; \( V_{FV}(t) = V_{FV} \) is the landing speed of UAV.

For given values of \( V_{FV} \) and \( \Delta l \) solution (1) is trivial and does not depend on the second part of the complete solution of the problem.

Suppose that the engagement by the break hook is performing in the middle of the rope. In this case \( F_A = F_B \) and each half of the rope will be stretched by a value

\[
\Delta l = \frac{\sqrt{L^2 + 4(\Delta l)^2} - L}{2}.
\]

The magnitude of the stretching of half the rope \( \Delta l \) according to Hooke’s law is determined as

\[
\Delta l = \frac{F_1}{SE \left( \frac{L}{2} \right)}, \quad \Delta l = \frac{F_1}{SE \left( \frac{L}{2} \right)},
\]

where \( F_1 \) is a tensile force of half the rope, \( F_1 = F_A = F_B \); \( S \) is a cross-sectional area of the rope, \( E \) is the Young’s modulus.

According to Newton’s 2nd law, the magnitude of the rope stretching force \( F = 2F_A \sin \alpha \), where \( \sin \alpha = \frac{\Delta l}{\sqrt{L^2 + (\Delta l)^2}} \), is determined by the product of the mass of the UAV on acceleration at braking, i.e.

\[
F = m_{FV} a.
\]

Equating the right-hand sides of equations (2) and (3) with account (4), we obtain an equation for the quantity \( L \) at \( \sin \alpha = \frac{\Delta l}{\sqrt{L^2 + (\Delta l)^2}} = \frac{\Delta l}{L} \) in the form

\[
\sqrt{L^2 + 4(\Delta l)^2} - L = \eta \frac{L^2}{\Delta l},
\]

where \( \eta = \frac{m_{FV} a}{4SE} \) is an accessory parameter.

After transformation (5) we can write this equation in the form

\[
\eta^2 \xi^4 + 2\eta \xi^3 - 4 = 0,
\]

where variable \( \xi = L / \Delta l \).
If we multiply (6) on \( \eta^2 \) and introduce the designation \( x = \eta \xi \), then receive
\[
x^4 + 2x^3 - 4\eta^2 = 0. \tag{7}
\]

Using the solution of Descartes–Euler with help by substitution \( x = y - 0.5 \), we reduce (7) to “incomplete” type
\[
y^4 - 1.5y^2 + y - \left(\frac{3}{16} + 4\eta^2\right) = 0. \tag{8}
\]

The roots of equation (8) equal one of the following expressions \( z_1, z_2, z_3 \) are roots of cubic
\[
z^3 - 0.75z^2 + \frac{C_1z - \frac{1}{64}}{4} = 0, \tag{9}
\]
where \( C_1 = \frac{3}{16} + \eta^2 \).

After replacement \( z = p + 0.25 \) and transformation (9) we receive
\[
p^3 + \eta^3p + \eta^2 \frac{4}{4} = 0. \tag{10}
\]

After Vieta’s replacing \( p = \omega - \eta^2 / (3\omega) \)
and transformation (10) we receive a biquadratic equation
\[
\varepsilon^2 + \frac{\eta^2\varepsilon}{4} - \frac{\eta^6}{27} = 0, \tag{11}
\]
where \( \varepsilon = \omega^3 \).

The roots of equation (11) are
\[
\varepsilon_{1,2} = -\frac{\eta^2}{8\left(1 \pm \sqrt{1 + 64\frac{\eta^2}{27}}\right)}. \tag{12}
\]

But \( x = \frac{\eta L}{\Delta l} = y - 0.5 > 0 \), then must be inequality \( y > 0.5 \), i.e. \( \sqrt{z} > 0.5 \) and therefore \( p > 0 \).
Numerical solution (6) at different meanings \( \eta \) (Fig. 3) indicates that: 1) at considered values of parameters for the UAV and the rope exists only the one decision (6); 2) it is necessary to take the absolute value \( x \) when we find a solution of the task from (8) – (12).

During boarding on the rope, the UAV will experience an overload, magnitude of which is equal
\[
n_i = \frac{m_{fr}a}{m_{fr}g} = \frac{a}{g}. \tag{13}
\]

Assuming that during the UAV brakes, its kinetic energy passes into the potential energy of the stretched rope, we can write
\[
\frac{m_{fr}V_f^2}{2} = K_T \frac{(2\Delta l)^2}{2}. \tag{14}
\]
where \( K_T \) is the coefficient of rigidity of the brake gates rope.

The value of \( K_T \) depends both on the properties of the material and on the dimensions of the rope.

Dependence of the value \( K_T \) from the rope dimensions and Young’s modulus may be express in the form
\[
K_T = \frac{ES}{L}. \]

Using (14) instead of (3) does not simplify a solution of the task.

Let consider case of UAV landing on rope, when \( AO \neq OB \). For calculation values of forces \( F_A \) and \( F_B \) we apply the principle of virtual displacements \[4\], as it shown in Fig. 4, where \( \theta \) is an angle of infinitesimal rotation round point A.
\[
\begin{align*}
-F_B \varepsilon + F_T \varepsilon_1 &= 0, \\
\varepsilon &= L \cdot \Delta \theta, \\
\varepsilon_1 &= l_0 \cdot \Delta \theta.
\end{align*}
\]

(15)

We receive from system (15) \( F_B = \frac{F_T l_0}{L} \).

Analogical we can receive the value \( F_A = F \frac{(L - l_0)}{L} \).

### IV. Calculation of the UAV parameters landing on rope

As an example, we will choose the UAV, which has a takeoff weight of 20.00 kg and a flight speed of 30 m/s. We choose the following changes of value \( \Delta l \) : in the range from 0.1 m to 1.5 m.

Solving system (1) and using equation (13) for given data, we will receive numerical values for \( t, a \) and \( n \), which are resulted in Table I. As follows from the calculations, the braking distance \( \Delta l = 0.1 \) m is insufficient for UAVs of this type since it is possible to destroy it at an overload of more than 458 units.

### Table I

<table>
<thead>
<tr>
<th>Parameters of the UAV motion</th>
<th>0.1</th>
<th>1</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t, s )</td>
<td>0.0067</td>
<td>0.067</td>
<td>0.1</td>
</tr>
<tr>
<td>( a, m/s^2 )</td>
<td>4500</td>
<td>450</td>
<td>300</td>
</tr>
<tr>
<td>( n )</td>
<td>458.72</td>
<td>45.87</td>
<td>30.58</td>
</tr>
</tbody>
</table>

Calculations were carried out for ropes with a cross section \( S = 8 \text{ cm}^2 = 8 \cdot 10^{-4} \text{ m}^2 \) (Table II), made of silk threads \((E_s = 13.0 \cdot 10^9 \text{ Pa})\) and of rubber \((E_r = 0.9 \cdot 10^9 \text{ Pa})\). Calculations showed that the required length of the rope made of silk threads is in several times longer than the rope made of rubber at the same value \( \Delta l \).

### Table II

<table>
<thead>
<tr>
<th>Rope material</th>
<th>Rubber</th>
<th>Silk thread</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta l, \text{ m} )</td>
<td>1.0</td>
<td>1.5</td>
</tr>
<tr>
<td>( \eta \cdot 10^4 )</td>
<td>31.25</td>
<td>20.83</td>
</tr>
<tr>
<td>( L, \text{ m} )</td>
<td>4.29</td>
<td>7.37</td>
</tr>
<tr>
<td>( \Delta l_0, \text{ m} )</td>
<td>0.222</td>
<td>0.293</td>
</tr>
</tbody>
</table>

### V. Conclusion

In the paper analytical expressions are obtained, which allow making an approximate calculation of parameters of the UAV landing on the rope without additional dampers. The above calculations showed that the required braking ropes, without additional cushioning devices (springs), may be used in practice. But, during landing on a rope the UAV can experience significant longitudinal overloads, therefore it is necessary carefully select the characteristics of rope.

**References**


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Н. Ф. Тупицин, І. В. Чичкань. Наближені обчислення характеристик процесу посадки БПЛА на трос
Отримано аналітичні вирази, що зв'язують параметри безпілотного літального апарату і характеристики троса, на який він здійснює посадку. Зокрема, визначається співвідношення між необхідною довжиною кабелю і значенням його величини розтягування під час посадки безпілотного літального апарату. При цьому, додаткові амортизатори для троса не розглядаються. Математичну модель процесу посадки безпілотного літального апарату на трос засновано на законі Гука і 2-му законі Ньютона. Одним з основних припущень під час розробки математичної моделі є те, що гальмування безпілотного літального апарату після його зчепки з тросом, відбувається з постійним у скороченням.

Ключові слова: посадка безпілотного літального апарату; розтягування каната; закон Гука; перевантаження безпілотного літального апарату.

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