OPTIMAL PATH OF NANOSATELLITES GROUP INJECTION FOR EARTH REMOTE MONITORING

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Abstract—The paper proposes the solution of model problem considering all occurrences and effects related with motion of multiple head rocket injecting the group of nanosatellites. The research led to determining the optimum path that can be used for orbital injection of group of nanosatellite multisensor-based platforms for the purpose of integrated study of Earth superstandard layer.

Index Terms—Multisensor-based platforms; nanosatellites; orbital injection; branching path.

I. INTRODUCTION

Beginning of the third millennium coincided with a new stage of development of technologies of miniature spacecraft: micro- and nano-satellites. Nanosatellites (nanosatellites, nanosats) are spacecraft with a mass from 1 kg to 10 kg, the size of 1U (10×10×10 cm), 2U (10×10×20 cm) and 3U (10×10×30 cm), designed to solve simple and important tasks. Nanosatellites will be used for remote sensing, environmental monitoring, earthquake prediction and study of the ionosphere [6], [7], [9].

The time of individual breakthroughs and first successful experiences of small satellites building is over. The main task today is the insertion of nanosatellites into orbit.

II. PROBLEM STATEMENT

Analysis of researches and publications. The analysis of optimal path determination for orbital injection of nanosatellites group, shows that the optimal control of systems was studied in researches of L. Ashepkov [1], E. Sage, H. White [2], the stochastic differential systems was analyzed by V. Pugachev, I. Sinitsyn. [3].

A problem of optimal control by determined complex dynamic systems moving along branching paths and allowing to inject the group of navigation satellites at once is not examined.

It is proposed to launch nanosatellites as a payload on the basis of An-124-100 airplane, which is used as a mobile launch pad to launch the SS-24 light class solid-propellant launch vehicle (LV) [8]. The airplane with LV placed in cargo compartment will take off from a conventional airfield and lift to a height of about 20 km, where with the help of air-launched including transporter-erector-launcher (TEL) and parachute system.

The launch vehicle located on the TEL starts to move on the floor equipment (rollers) under the parachute forces towards the cargo door as a result of preparations for the dropping operations related to cargo door opening, activation of control systems, etc. The belts binding the LV and TEL or other devices are disconnected at the moment of physical separation from the airplane.

Using own solid-propellant engine (during the initial stage of the flight) and from force of inertia afterwards the LV with the launch container including nanosatellites, goes to a height of about 600 km which is targeted for dropping a payload in the form of miniature satellites. Following this the TEL lands by means of parachute in a predetermined position and is ready for further (multiple) use. The nanosatellites are installed on the platform inside the container and are pressed to the cover by springs.

The electric pulse from the LV activates the release mechanism of the cover that rotates 170° and a nanosatellite is separated along the guide rails by spring mechanism to predetermined speed, which is determined by the used spring and nanosatellite weight. A small payload is separated due to the magnetic-impulse drive with capacitive energy storage.

The proposed project is best suited to existing and other potential limitations owing to launching from airplane using customers’ aerodromes and air space.

III. PROBLEM SOLUTION

The active area of motion path of rocket that injects the group of nanosatellites taking into account the exact model of multiple head rocket motion, model of Earth's atmosphere and terrestrial gravitational field can be optimized only at the successful first approach to the optimal solution [6], [7]. The solution of the model problem laid down in the article can be accepted for the first approach (suboptimum solution) for more detail problem considering more precisely all occurrences and effects related to multiple head rocket that injects group of navigation satellites.
We consider the following statement of problem. Suppose, the motion of LV with multiple heads is described by the differential system [4], [5].

\[ \dot{x} = f(x, u, t), t \in [t_0, t_f], x \in E^n, u \in \Omega \subset E^m, \]

where \( x, u \) are vectors of phase state and control actions influencing on LV motion; \( t_0, t_f \) are instants of LV motion start and stop at the given range.

Assume that the LV head includes \( k \) rockets (total \( \sum_{i=1}^{k} r_i \)). The rockets are separated \( k \)-times from LV head (Fig. 1). The LV starts controllable motion at point 0 and moves along branch from point 0 to point 1. Next the rockets moving from point 1 to points 11 and 12 are separated.

![Diagram of path of LV with multiple heads](image)

**Fig. 1.** The diagram of path of LV with multiple heads

The launch vehicle starts according to the variety shown on Fig. 1

\[ g_{i}^{(0)}(x_i(t_0), t_0) \begin{cases} = 0, l = 1, l^{(0)}_g; \\ \leq 0, l = k^{(0)}_g + 1, n^{(0)}_g. \end{cases} \]

(2)

According to the varieties

\[ g_{i}^{(0)}(x_i(t_i), t_i) \begin{cases} = 0, l = 1, l^{(0)}_g; \\ \leq 0, l = k^{(0)}_g + 1, n^{(0)}_g, \end{cases} \]

(3)

the separation of heads \( r_i = 1/k \) occur. The heads are moved to the varieties

\[ g_{i}^{(0)}(x_i(t_j), t_j) \begin{cases} = 0, l = 1, l^{(0)}_g; \\ \leq 0, l = k^{(0)}_g + 1, n^{(0)}_g, \end{cases} \]

(4)

where these are stop to inject the nanosatellites group.

The dynamic equations of separable heads motion are [4], [5]

\[ \dot{x}_\beta = f_\beta(x_\beta, u_\beta, t), t \in [t_y, t_p], \]

\[ x_\beta \in E^n, u_\beta \in E^{m_\beta} \in \Omega_\beta, (\beta = 1, 11, 12), \]

\[ (\beta = 1, \beta^* = 0; \beta = 11, 12, \beta^* = 1), t_0 < t_1 < t_{12} < t_{11}. \]

The following constraints are applied on them

\[ Q_i^{(1)}(x_i(t), u_i(t), t) \begin{cases} = 0, l = 1, K_{i}^{(1)}; \\ \leq 0, l = K_{o}^{(i)} + 1, N_{o}^{(i)}; \end{cases} \]

(6)

\[ Q_{i1,12}^{(1,1,2)}(x_{i1,12}(t), u_{i1,12}(t), t) \begin{cases} = 0, l = 1, K_{o}^{(1,1,2)}; \\ \leq 0, l = K_{o}^{(1,1,2)} + 1, N_{o}^{(1,1,2)}; \end{cases} \]

(7)

\[ Q_i^{(1,1)}(x_i(t), u_i(t), t) \begin{cases} = 0, l = 1, K_{i}^{(1)}; \\ \leq 0, l = K_{o}^{(i)} + 1, N_{o}^{(i)}; \end{cases} \]

(8)

The following conditions should be met at the separation moments

\[ x_i(t_i) - x_j(t_i) = 0 (i = 1, \overline{k}; j = 1, \overline{r}), \]

\[ x_i(t_i) - x_{i1-i2}(t_i) = 0 (i = 1, k-1) \]

(9)

for all the phase coordinates except for coordinate describing the main head mass variation, for which (let it be \( n \) coordinate) the condition is satisfied

\[ x_n(t_i) = \xi(i)x_{i1-i2}(t_i) + \sum_{j=1}^{r} x_{j}(t_i), \]

\[ i = 1, \overline{k}; \quad j = 1, \overline{r}, \quad \xi(i) = \begin{cases} 1, i = 1, k-1; \\ 0, i = k. \end{cases} \]

(10)

Equation \( u_\beta(t) \), phase coordinates \( x_i(t_0), x_\beta(t_p) \), and instants \( t_0, t_\beta (\beta = i, j), (i = 1, k; j = 1, \overline{r}) \) should be selected so that to minimize the criterion [5]

\[ I = S(x_i(t_0), t_0; x_i(t_i), t_i; x_{i1}, t_{i1}); \]

\[ x_{i1}(t_{12}), t_{12}; x_{i1}(t_{i1}), t_{i1} + I_{1} + I_{11} + I_{12} \rightarrow \min \]

\[ x_{i1}(t_{i1}), t_{i1} \begin{cases} = 0, l = 1, K_{o}; \\ \leq 0, l = K_{o} + 1, N_{o}; \end{cases} \]

(11)

\[ x_i(t_i) - x_{i1}(t_i) = 0, x_i(t_i) - 12x_i(t_i) = 0 (\tau = 1, n-1), \]

\[ x_n(t_i) - 11x_n(t_i) - 12x_n(t_i) = 0. \]

Thus, the nanosatellites path optimization task is to find the optimal equations, LV path (1) – (4) and
separable heads along branching path segments (5) – (10) minimizing the criterion (11) as well to discover the optimal instants and phase coordinates when the separation occurs.

The specified task is assumed to be solved in three stages:

1) The diagram of branching path is drawn based on physical conditions, the equations describing LV and separable heads motion along path branches are set up.

2) The chronological sequence of heads from LV separation instants are set up.

3) The augmented vectors of state and control are established on the basis of minimum principle. These include the state and control vectors of separated heads moved along path branches at the present time space and satisfy optimal task conditions (1) – (4) to specify optimal path of main head.

Such approach leads to the following problem statement. The rocket with two-part head starts the controlled motion in a point 0 and moves further along the branch 0-1 to the point 1.

Then the head is divided into parts. The parts move from the point 1 to the points 11 and 12 by own controls. We suppose that motion takes place in a vacuum in the plane-parallel gravitational field.

The diagram of branching path of multiple head rocket injecting a group of navigation satellites is shown in Fig. 2.

![Fig. 2. The diagram of branching path](image)

The equations describing movement of rockets on the relevant path sections have the forms:

\[
\dot{q}_1 = g_{q_1} \cos q_v, \\
\dot{q}_2 = q x_3, \\
\dot{q}_3 = g_{q_3} \sin q_v - g, (q = 1, 11, 12),
\]

where \(\dot{q}_1\), \(\dot{q}_3\) are velocity components directed to across and along of force lines of gravitational field; \(q x_3\) is the moving coordinate, counted along force lines of gravitational field (flight altitude); \(P_q\) is the acceleration created by rocket power plant injecting the group of nanosatellites on the segment 0-1 of the path \((q=1)\) and by power plant of separating rockets on the segment 1-11 \((q=11)\) and 1-12 \((q=12)\); \(q_v\) is the pitch angle; \(g\) is the gravitational acceleration.

We consider the problem with fixed instants: \(t_0\) is the start of motion of the rocket injecting the group of nanosatellites; \(t_1\) is the head separation; \(t_{11}\) and \(t_{12}\) goal achievement by the separated rockets goal achievement by the separated rockets.

Initial position of the rocket injecting the group of navigation satellites, \(q_1(t_0), \ q_2(t_0), \ q_3(t_0)\) is predetermined.

The acceleration \(P_q(q=1, 11, 12)\) on the each segment of the path is constant.

It is necessary to determine the law of coordinates variation \(v_q(q=1, 11, 12)\) of point 1, for which the criterion

\[
I = -\left[11x_2(t_{11}) + 12x_2(t_{12})\right]
\]

reaches a minimum provided that \(11x_1(t_{11}), \ 11x_3(t_{11}), \ 12x_1(t_{12}), \ 12x_3(t_{12})\) are predetermined.

It is possible to define the required conditions of the control optimality \(v(q) \in [t_0, t], \ 11v(t) \in [t, t_{11}], \ 12v(t) \in [t, t_{12}]\) on the basis of the principle of a minimum for the component dynamic systems [5] equations (12) – (15): to determine the optimal branching path of rocket injecting the group of nanosatellites, it is necessary to find the constant variables

\[
\begin{align*}
\dot{\lambda}_1 &= -\frac{\partial H_q}{\partial q x_1}, \\
\dot{\lambda}_2 &= -\frac{\partial H_q}{\partial q x_2}, \\
\dot{\lambda}_3 &= -\frac{\partial H_q}{\partial q x_3},
\end{align*} \quad (q=1, 11, 12),
\]

where

\[
H_q = g_{q_1}(1)P_q \cos q_v + g_{q_3}(1)x_3 + g_{q_3}(1)P_q \sin q_v - g_{x_3}(1)g,
\]

satisfying the conditions:

\[
\begin{align*}
\frac{\partial I}{\partial 11x_2(t_{11})} - 11\dot{\lambda}_2(t_{11}) &= \frac{\partial I}{\partial 12x_2(t_{12})} - 12\dot{\lambda}_2(t_{12}) = 0, \\
\frac{\partial I}{\partial 11x_1(t_{11})} - 11\dot{\lambda}_1(t_{11}) + 11\dot{\lambda}_1(t_{11}) + 12\dot{\lambda}_1(t_{11}) &= 0,
\end{align*}
\]
\[
\frac{\partial I}{\partial x_i(t)} - \lambda_2(t) + \lambda_2(t) + \lambda_2(t) = 0, \quad (20)
\]

\[
\frac{\partial I}{\partial x_i(t)} - \lambda_3(t) + \lambda_3(t) + \lambda_3(t) = 1
\]
at arbitrary values \(\lambda_1(t_{i1}), \lambda_1(t_{i2}), \lambda_1(t_{i1}), \lambda_1(t_{i2}), \lambda_1(t_{i1}), \lambda_1(t_{i2})\), in order to minimise a Hamiltonian on control \(q\) \((q = 1, 11, 12)\).

\[
q \dot{x}_i(t) = \frac{g P_q}{A_q} \left[ 1 - \frac{1}{4C_q} \left( 2C_q t + b_q \right) \sqrt{R_q(t)} \left( 2C_q t^* + b_q \right) \sqrt{R_q(t^*)} \right] + \frac{4A_q^2}{8C_q^2} q \dot{x}_i(t^*) - g \left( t - t^* \right)^2 + q \dot{x}_i(t^*) + \dot{x}_i(t^*), \quad (24)
\]

where

\[
R_q(t) = a_q + b_q C_q t^2, \quad a_q = 1 + B_q^2, \quad b_q = 2A_q B_q,
\]

\[
C_q = A_q^2 \left( q = 1, 11, 12 \right);
\]

if

\[
q = 1, t \in \left[ t_0, t_f \right], t^* = t_0, A_q = 2 \left( t_{i1} + 1 + t_{i1} \right)^{-1},
\]

if

\[
B_q = \left[ t_{i1} \lambda_3(t_{i1}) + t_{i2} \lambda_3(t_{i2}) - 2t_{i1} \right] \left( t_{i1} + 1 + t_{i1} \right)^{-1},
\]

if

\[
q = 11, t \in \left[ t_1, t_{i1} \right], t^* = t_1;
\]

if

\[
q = 12, t \in \left[ t_1, t_{i2} \right], t^* = t_1;
\]

\[
A_q = \lambda^{-1}_3, B_q = \left[ \lambda^{-1}_3(t_1) - t_1 \right] \lambda^{-1}_3 \left( q = 11, 12 \right).
\]

The parameters \(\lambda_1, \lambda_1, \lambda_1(t_{i1}), \lambda_1(t_{i2})\) should be selected so as to meet the required final conditions.

The results of optimal branching path calculation are shown in Fig. 3 and 4 for following initial data:

\[
t_0 = 0 s, \ t_1 = 70 s, \ t_{i1} = 100 s, \ t_{i2} = 120 s,
\]

\[
\dot{x}_i(t_0) = \dot{x}_i(t_0) - \dot{x}_i(t_0) = 0 m/s, \ \dot{x}_i(t_0) = 0 m/s,
\]

By applying the relations (16) – (21) for the task (12) – (15), we receive the solution in an explicit analytical form:

\[
q \dot{x}_i(t) = \frac{g P_q}{\sqrt{C_q}} \left[ \left( 2C_q R_q(t) + 2C_q^2 t + b_q \right) \left( 2C_q R_q(t^*) + 2C_q^2 t^* + b_q \right) \right] \cdot \frac{4A_q^2}{8C_q^2} q \dot{x}_i(t^*)
\]

+ \dot{x}_i(t^*) - \dot{x}_i(t^*) + \dot{x}_i(t^*)
\]

Fig. 3. Graphs of optimal branching path parameters – altitude and speed: \(v_q = \sqrt{v_1^q + v_2^q} \quad (q = 1, 11, 12)\),

The parameters values \(\lambda_1, \lambda_1, \lambda_1(t_{i1}), \lambda_1(T(t_{i2}))\) have been found by solving four nonlinear equations (22), (24) with use the gradient method at \(q = 11, 12\) with due account for finite conditions (26) and (27). The values \(\lambda_1(t_{i1}) = -50 s, \lambda_1(t_{i1}) = -30 s\) are used as first approximation.

The calculations have been stopped at \(\lambda_1(t_{i1}) = -100.06 s, \lambda_1(t_{i1}) = -49.00 s, \lambda_1(t_{i1}) = -26.60 s, \lambda_1(t_{i1}) = -33.26 s\), when the permissible errors of final conditions have reached the values

\[
\Delta_{11}x_1(t_{i1}) = 1.049 m/s, \ \Delta_{11}x_1(t_{i1}) = 0.268 m/s,
\]

\[
\Delta_{12}x_1(t_{i1}) = 1.34 m/s, \ \Delta_{12}x_1(t_{i1}) = 0.510 m/s
\]
$$x_2(t_1) = 124.889 \text{ km}, \quad x_2(t_{11}) = 131.834 \text{ km},$$
$$x_2(t_{12}) = 162.795 \text{ km}$$

Fig. 4. Graphs of optimal branching path parameters—optimal control: $tg_1(v(t_1) = 1.34$; $tg_1(v(t_{11}) = 0.401$;

$$tg_{11}(v(t_2) = 0.265$; $tg_{12}(v(t_2) = 0.678$; $v(t_5) = 53^016'$$;

$$v(t_6) = 21^051'$; $v(t_7) = 14^051'$$; $v(t_8) = 34^09'$

The optimal value of criteria (15) equals $I = -294629$ m.

IV. CONCLUSION

Hence, the paper proposes mathematical formulation and search task solution of optimal multiple head rocket path as well optimal instants and phase coordinates when the heads with Earth monitoring nanosatellites group are separate.

The resulting control affects at various stages of flight and at the moment of separation allow to use effectively the resources of compound dynamic system for optimal path of nanosatellites group injection into orbit round the Earth.

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Received May 12, 2016

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О. М. Тачиніна. Оптимальна траєкторія виведення групи наноспутників для дистанційного моніторингу Землі

Запропоновано рішення моделюваної задачі, що враховує всі явища і ефекти, пов’язані з рухом ракети, головна частина якої розділяється і виводить групу наноспутників. В результаті проведених досліджень отримано оп-
Точная траектория, которая может использоваться для выведения на орбиту группы наноспутников для дистанционного мониторинга Земли.

**Ключевые слова:** мультисенсорная платформа; наноспутники; выведение на орбиту; ветвящиеся траектории.

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Кількість публікацій: 85

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