I. INTRODUCTION

The flight dynamics of a modern aircraft are highly nonlinear and vary over the wide flight envelope. This fact makes it difficult for a single controller to achieve the desired closed-loop specifications. This paper considers gain-scheduling successive loop control design for parameter-varying system in terms of linear matrix inequalities. The obtained controller guarantees an efficient unmanned aerial vehicle’s control under external disturbances within the flight envelope. The design procedure is illustrated by a case study of unmanned aerial vehicle lateral channel control.

Index terms—Gain scheduling; linear matrix inequalities; robustness; unmanned aerial vehicle.

II. ANALYSIS OF RECENT RESEARCH AND PUBLICATIONS

The gain scheduling control problem has been widely developed both from theoretical and practical viewpoints, see, for example, [11], [12]. An effective approach to solve the nonlinear control problem is using gain scheduling with linear parameter-varying (LPV) controller [18]. For example, the parameters of the mathematical model of an aircraft depend on the altitude and speed (Mach number), which determine the dynamic pressure (DP):

\[
\bar{q} = \rho v^2 / 2,
\]

where \( \rho \) is density of air, kg/m\(^3\), \( v \) is air speed, m/s. The main advantage of DP, that this parameter is related with both flight altitude and speed values. As far as all entries of the stability and control matrices \( A, B \) could be considered as functions of \( \bar{q} \): \( A(\bar{q}), B(\bar{q}) \) [13], then for these matrices it is possible to find parameters of the controller from the point of view of robust stability and robust performance for each numerical value of \( \bar{q} \).

III. PROBLEM STATEMENT

Let us consider a LPV system in form

\[
\begin{align*}
\dot{x}(t) &= A(\bar{q})x(t) + B(\bar{q})u(t) + B_e(\bar{q})\nu(t), \\
y(t) &= C(\bar{q})x(t) + D(\bar{q})u(t),
\end{align*}
\]  

where \( x = [\beta, p, r, \varphi, \psi, \varphi]^T \) is the state vector with components: sideslip angle \( \beta \), roll and yaw rates \( p, r \) respectively, and roll and yaw angles \( \varphi, \psi \) and cross-track error \( \bar{y} \); \( u = [\delta_{s}, \delta_{r}]^T \) - the control vector with components: the deflection of ailerons and rudder respectively; \( y = [p, r, \varphi, \psi, \bar{y}, V_y]^T \) is the observation vector, where \( V_y \) stands for the side velocity; \( A, B, C \) and \( D \) are the state matrices that depend on parameter DP; \( B_e \) is the matrix external disturbance.

The block diagram of the UAV lateral motion control system with SLP is shown in Fig. 1 [19].

![Fig. 1. Block diagram of the UAV lateral motion with successive loop control](http://ecs.in.ua)
It represents 3 successive closed loops with standard PD control laws, so the innermost loop (roll angle control) has control law
\[ \delta a(z) = W_{ac}(z) \left( K_{o \theta} + K_{p} p \right). \] (2)

The same PD control laws are accepted for the yaw angle control (intermediate loop) with coefficients \( K_{o \gamma}, K_{\gamma} \) as well as for the outermost cross-track error \( y \) loop with coefficients \( K_{z}, K_{\nu} \).

The output of the outer control law serves as the reference (command) signal to the corresponding inner loop. It is known \([2, 8]\) that in order to suppress sideslip angle \( \beta \) for the coordinated turn execution, the standard washout filter with transfer function
\[ W_{wf}(s) = K_{wf} \frac{T_{wf}s}{T_{wf}s + 1}, \]
is applied as a local feedback from the yaw rate \( r \) sensor to the deflection of rudder \( \delta_{r} \). For the sake of the further simplification, we neglect the dynamics of this local loop, so we will consider only main contour with single input – deflection of the aileron \( \delta_{a} \), as it is shown in the Fig. 1.

The goal of the research is to design a family of local LMI-controllers.

The algorithm of controller design in terms of LMIs was proposed in \([5]–[7]\).

The control law is given by
\[ u(t) = -K_{y}v(t) = -KC_{x}(t), \] (3)

where \( K \) is a constant output feedback gain matrix. The exogenous disturbances \( v \) are restricted by \( L_{2} \)-norm
\[ \|v(t)\| = \int_{0}^{\infty}(v^{T}v)dt < \infty. \] (4)

\( L_{2} \)-norm assures disturbance attenuation with a predefined level, \( \gamma \).

\( K \) matrix minimizes a performance index:
\[ J(K) = \int_{0}^{\infty} \|z(t)\|^{2} dt = \int_{0}^{\infty} (x^{T}Qx + u^{T}Ru)dt \]
\[ \leq \gamma^{2} \int_{0}^{\infty} v^{T}vdtd, \quad \forall v(t) \neq 0, \]

where \( Q \geq 0, \quad R > 0 \) are diagonal matrices, weighting each state and control variables, respectively. Output signal \( z(t) \) used for performance evaluation is defined as follows:

\[ z = \begin{bmatrix} \sqrt{Q} & 0 \\ 0 & \sqrt{R} \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix}. \]

The system \( L_{2} \) gain is said to be bounded or attenuated by \( \gamma \) if \([3, 5, 6, 10]\):
\[ \gamma^{2} \leq \frac{1}{\gamma^{2}} \int_{0}^{\infty} (x^{T}Qx + u^{T}Ru)dt \]
\[ = \frac{1}{2} \int_{0}^{\infty} \|v(t)\|^{2} dt \]
\[ \leq \gamma^{2} \int_{0}^{\infty} v^{T}vdtd, \quad \forall v(t) \neq 0, \]

where \( i = 1, ..., N \) in (5) denotes the set of models associated with scheduled operating conditions within the flight envelope.

The matrices \( K \) are:
\[ K_{i} = R^{-1}B_{i}^{T}P_{i}C_{i}^{T}(C_{i}C_{i}^{T})^{-1}. \]

It is desired to find a family of static output feedback control gain matrices \( K \) such that the system is stable and the \( L_{2} \) gain is bounded by a prescribed value \( \gamma \).

IV. GAIN SCHEDULING CONTROLLER

A gain scheduling control system design takes following steps:
1. Choose the operating points or region in the scheduling space, which is defined by flight envelope of UAV. Obtain a plant model for each operating region by linearizing the plant’s model in the several equilibrium operating points.
2. Design a family of local LMI-controllers for the obtained plant models.
3. Implement a scheduling mechanism.
4. Assess the GS closed loop stability and performance.

The block-scheme of a GS-feedback loop is shown on Fig. 2.

The vector of adjustable parameters of the autopilot \( K \) has the following components:
\[ K(q) = [K_q(q), K_p(q), K_v(q), K_r(q)]' \]

(6)

The objective of linearization scheduling is that the equilibrium family of the controller (6) matches the equilibrium family of the plant (1), such that:
- the closed-loop system still can be tuned appropriately with respect to performance and robustness demands;
- the linearization family of the controller equals the designed family of linear controllers.

The variable \( b \) represents the aircraft wingspan. The variables \( \sigma_v, \sigma_w \) represent the turbulence scale lengths. The variables \( \sigma_v, \sigma_w \) represent the turbulence intensities.

The performance and robustness indices are possible to estimate after a family of gain-scheduled static output controllers is obtained using proposed approach. Thus, performance is estimated by \( H_2 \)-norm of system function with respect to disturbance, whereas the robustness is estimated by \( H_\infty \)-norm of the complementary sensitivity function [16].

1. \( H_2 \)-norms of system sensitivity function in deterministic case:

\[ \| H \|_{2\text{det}} = \sqrt{\text{trace}(C_W C_s^{-1})} \]

where \( W \) is a controllability gramian and \( C_s \) is a weighting matrix in deterministic.

2. \( H_2 \)-of system sensitivity function in stochastic case:

\[ \| H \|_{2\text{st}} = \sqrt{\text{trace}(C_W C_s^{-1})} \]

where \( W \) is a controllability gramian and \( C_s \) is weighting matrix in stochastic case.

3. \( H_\infty \)-norm of the complementary sensitivity function:

\[ \| H(j\omega) \|_\infty = \sup_{\omega} \sigma(H(j\omega)) \]

where \( \sigma \) is the singular value of complementary sensitivity matrix; \( \omega \) is the maximum singular value on the current frequency.

IV. CASE STUDY

The block-diagram of the closed-loop system for control of lateral motion is depicted on Fig. 3, where \( \eta \) is white noise vector, \( y_{ref} \) is cross track distance reference signal, \( h_{ADC} \) and \( V_{ref} \) are altitude and true speed measured by Air Data Computer.

The nonlinear model of the Aerosonde is linearized for the range of operating conditions respected to the range of DP from 200 to 650 kg/(ms

The state space matrices \( A \) and \( B \) in general form filled with stability and control derivatives are given below [17]:

\[
A = \begin{bmatrix}
Y_p & Y_p & \frac{Y_p}{V} & \frac{g \cos \theta_v}{m} & 0 & 0 \\
L_p & L_p & L_v & 0 & 0 & 0 \\
N_v & N_v & N_p & 0 & 0 & 0 \\
0 & \frac{\cos \theta_v}{\sin \theta_v} & \frac{\sin \theta_v}{\cos \theta_v} & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
V_t & 0 & 0 & 0 & V_t & 0 \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
Y_{ba} \\
L_{ba} \\
N_{ba} \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

where \( Y_p = \frac{\bar{q} S}{m} C_{Y_p}; \) \( Y_p = \frac{\bar{q} S b}{2mV_t} C_{Y_p}; \) \( Y_p = \frac{\bar{q} S b}{2mV_t} C_{Y_p}; \)
\[ Y_{ha} = \frac{\bar{q} S}{m} C_{h_a}; \quad L_p = \frac{\bar{q} S b}{J_x} C_{i_p}; \quad L_p = \frac{\bar{q} S b}{J_x} C_{i_p}; \]
\[ L_x = \frac{\bar{q} S b}{J_x} C_{i_r}; \quad L_r = \frac{\bar{q} S b}{J_x} C_{i_r}; \quad L_{ha} = \frac{\bar{q} S b}{J_x} C_{i_{ha}}; \]
\[ L_p = \frac{\bar{q} S b}{J^*_z} C_{i_p}; \quad N_p = \frac{\bar{q} S b}{J^*_z} C_{i_{p}}; \quad N_r = \frac{\bar{q} S b}{J^*_z} C_{i_{r}}; \]
\[ \theta = \theta_e, \quad \gamma = \gamma_e \] are equilibrium (steady-state) conditions, \( J_x, J_z \) are moments of inertia.

As seen from the description of space matrices coefficients, the aircraft flight dynamic depends on \( DP \) value. The LPV controller model is a finite set of linear controller models obtained for the operating grid of \( DP \) values. The set of linear controllers are shown in Table I. Linear interpolation on a set of data points \( (K_{i_{ha}}, q_{i}), (K_{i_r}, q_{i}), (K_{i_r}, q_{i}), (K_{i_{ha}}, q_{i}), (K_{i_p}, q_{i}) \) is defined as the concatenation of linear interpolants between each pair of data points.

Table II reflects standard deviations of the UAV outputs in a stochastic case of parametrically perturbed model with the static output feedback controller in a control loop.

As it follows from this table r.m.s. of state variables in the stochastic case are varying in reasonable limits. Performance and robustness indices are shown in Table III.

The low variation of the values of \( H_{\infty} \)–norms proves the high degree of the system robustness. Basing on the results of the \( H_2 \)-norm values, it is possible to conclude that the norms vary in small ranges. Their close values give a possibility to state that the efficiency of the closed-loop system is held at the desired level.

To illustrate the applicability of our approach, we present numerical examples.

---

**Fig. 3. Block diagram of the UAV lateral motion**

**Table I**

<table>
<thead>
<tr>
<th>( DP, \text{kg} \cdot \text{m}^2 \text{s}^{-2} )</th>
<th>Controller gains ( \bar{K}(q) = [K_{i_{ha}}(q), K_{i_r}(q), K_{i_r}(q), K_{i_r}(q), K_{i_p}(q), K_{i_{ha}}(q), K_{i_{ha}}(q)]^T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(less than 200)</td>
<td>[0.0762, 5.5182, 2.8793, 1.4693, 0.1947, 1.162];</td>
</tr>
<tr>
<td>250</td>
<td>[0.0778, 5.4542, 3.0032, 1.3543, 0.186, 1.1596];</td>
</tr>
<tr>
<td>300</td>
<td>[0.0774, 5.8683, 3.038, 1.3825, 0.1042, 1.0816];</td>
</tr>
<tr>
<td>350</td>
<td>[0.0802, 6.997, 3.7347, 1.498, 0.1626, 1.1449];</td>
</tr>
<tr>
<td>400</td>
<td>[0.0876, 6.727, 3.0362, 1.2665, 0.192, 1.1604];</td>
</tr>
<tr>
<td>450</td>
<td>[0.0897, 5.9078, 2.9427, 1.85, 0.1609, 1.144];</td>
</tr>
<tr>
<td>500</td>
<td>[0.0921, 4.431, 2.971, 1.5006, 0.2052, 1.146];</td>
</tr>
<tr>
<td>550</td>
<td>[0.0904, 6.284, 3.2591, 1.3928, 0.190, 1.1489];</td>
</tr>
<tr>
<td>600</td>
<td>[0.1069, 8.9126, 3.379, 1.345, 0.3529, 1.1508];</td>
</tr>
<tr>
<td>650 and above</td>
<td>[0.103, 9.2528, 3.1936, 1.379, 0.342, 1.1492];</td>
</tr>
</tbody>
</table>
The flight condition 1: flight altitude is 200 m, speed is 19 m/s, dynamic pressure is 216 kg·m/s².

The flight condition 2: flight altitude is 2050 m and speed is 30 m/s, dynamic pressure is 478 kg·m/s².

The flight condition 3: flight altitude is 1460 m, speed is 35 m/s, dynamic pressure is 651 kg·m/s².

Transient processes in nominal and parametrically perturbed system, which were simulated taking into account the influence of the random wind, for the input step function $\dot{y}_{ir} = 20$ m are shown in Fig. 4.

**TABLE II**

<table>
<thead>
<tr>
<th>DP, kg·m/s²</th>
<th>$\sigma_\rho$, deg</th>
<th>$\sigma_\rho$, deg/s</th>
<th>$\sigma_u$, deg/s</th>
<th>$\sigma_u$, deg</th>
<th>$\sigma_\omega$, deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.0892</td>
<td>0.1214</td>
<td>0.0206</td>
<td>0.0546</td>
<td>0.1873</td>
</tr>
<tr>
<td>250</td>
<td>0.0873</td>
<td>0.1255</td>
<td>0.0191</td>
<td>0.0519</td>
<td>0.1964</td>
</tr>
<tr>
<td>300</td>
<td>0.0727</td>
<td>0.1442</td>
<td>0.0354</td>
<td>0.0958</td>
<td>0.4691</td>
</tr>
<tr>
<td>350</td>
<td>0.09</td>
<td>0.1675</td>
<td>0.0253</td>
<td>0.0836</td>
<td>0.3326</td>
</tr>
<tr>
<td>400</td>
<td>0.0694</td>
<td>0.1678</td>
<td>0.0407</td>
<td>0.1272</td>
<td>0.4528</td>
</tr>
<tr>
<td>450</td>
<td>0.1079</td>
<td>0.154</td>
<td>0.0218</td>
<td>0.0305</td>
<td>0.0944</td>
</tr>
<tr>
<td>500</td>
<td>0.1094</td>
<td>0.1653</td>
<td>0.0255</td>
<td>0.0467</td>
<td>0.1199</td>
</tr>
<tr>
<td>550</td>
<td>0.1017</td>
<td>0.1461</td>
<td>0.0239</td>
<td>0.0404</td>
<td>0.1105</td>
</tr>
<tr>
<td>600</td>
<td>0.1108</td>
<td>0.2141</td>
<td>0.0277</td>
<td>0.0674</td>
<td>0.1735</td>
</tr>
<tr>
<td>650</td>
<td>0.108</td>
<td>0.2137</td>
<td>0.0269</td>
<td>0.0612</td>
<td>0.1656</td>
</tr>
</tbody>
</table>

**TABLE III**

<table>
<thead>
<tr>
<th>DP, kg·m/s²</th>
<th>$H_2$-norm Deterministic case</th>
<th>$H_2$-norm Stochastic case</th>
<th>$H_\infty$-norm</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;200</td>
<td>0.0437</td>
<td>0.1135</td>
<td>0.566</td>
</tr>
<tr>
<td>250</td>
<td>0.1028</td>
<td>0.3901</td>
<td>0.8288</td>
</tr>
<tr>
<td>300</td>
<td>0.2384</td>
<td>0.8646</td>
<td>0.8723</td>
</tr>
<tr>
<td>350</td>
<td>0.0769</td>
<td>0.3586</td>
<td>0.7574</td>
</tr>
<tr>
<td>400</td>
<td>0.2458</td>
<td>0.5277</td>
<td>0.6940</td>
</tr>
<tr>
<td>450</td>
<td>0.017</td>
<td>0.0271</td>
<td>0.6226</td>
</tr>
<tr>
<td>500</td>
<td>0.0457</td>
<td>0.0269</td>
<td>0.5774</td>
</tr>
<tr>
<td>550</td>
<td>0.0454</td>
<td>0.016</td>
<td>0.5299</td>
</tr>
<tr>
<td>600</td>
<td>0.0192</td>
<td>0.0314</td>
<td>0.599</td>
</tr>
<tr>
<td>650&gt;</td>
<td>0.0184</td>
<td>0.0185</td>
<td>0.6037</td>
</tr>
</tbody>
</table>

**Performance Indices**

The flight condition 1: flight altitude is 200 m, speed is 19 m/s, dynamic pressure is 216 kg·m/s².

$$A_1 = \begin{bmatrix} -0.532 & 2.234 & -18.867 & 9.719 & 0 & 0 \\ -3.471 & -17.0531 & 8.207 & 0 & 0 & 0 \\ 0.562 & -2.212 & -0.8578 & 0 & 0 & 0 \\ 0 & 1 & 0.1184 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 19 & 0 & 0 & 0 & 19 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -0.854 \\ -75.009 \\ -2.9575 \end{bmatrix}.$$
Fig. 4. Simulation results for lateral channel of the UAV in the presence of external disturbances (flight condition 1 is the solid line; the flight condition 2 is dash line; the flight condition 3 is dot line): (a) is bank angle; (b) is cross-track error; (c) is heading angle; (d) is roll rate; (e) is side velocity; (f) is yaw rate

From the shown below graphs it is evident that results are quite satisfying. Deflections of UAV angular characteristics are possible from the practical point of view. The cross-track value is held at the reference signal $y_{ref} = 20$ m with acceptable deflections. These figures along with numerical results, represented in Tables II, III show that desired robustness-performance trade-off is achieved. It can be seen that the handling quality of the nominal and the perturbed models are satisfied.

V. CONCLUSION

The work presented in this paper was motivated by an UAV with a wide flight envelope. Such vehicles have large parametric variations in the presence of uncertainties. The article presents a procedure of robust GS controller design. The flight control system consists of a SLC baseline controller which can be obtained conveniently by solving LMIs. The method is relatively easy to apply owing to the availability of UAV’s computational tools.

The flight envelope of the UAV refers to the capabilities of operating ranges in terms of speed and altitude. The dynamic pressure as function of altitude and speed was proposed as a simple gain scheduled tool for controller design.

The effectiveness of the proposed methods for designing flight controllers is verified and validated.
through the dynamic model in Matlab Simulink environment of the Aerosonde UAV. However, the proposed methods are also applicable to a general class of conventional aircrafts.

ACKNOWLEDGMENT

In addition, author highly appreciates Professor A. A. Tunik in helping to provide the research.

REFERENCES


Received February 09, 2016

Nadsadna Olga. Postgraduate student.
Aircraft Control Systems Department, National Aviation University, Kyiv, Ukraine.
Engineer-researcher ANTONOV Company, Kyiv, Ukraine.
Education: National Aviation University, Kyiv, Ukraine (2012).
Research area: control theory and its application.
Publications: 12.
E-mail: nadsadna@ukr.net

O. I. Надсадна. Робастна система керування беспілотним літальним апаратом з програмним забезпеченням коефіцієнтів посилення

Льотна динаміка сучасного літака, яка характеризується високим рівнем неелінійності і варіюється в широкому робочому діапазоні експлуатації об’єкта, ускладнює завдання забезпечення необхідних характеристик зворотного зв’язку з простим регулятором. Розглянуто синтез регулятора з програмним забезпеченням коефіцієнтів посилення для системи із змінними параметрами за допомогою апарату лінійних матричних нерівностей.
О. И. Надсадная. Робастная система управления беспилотным летательным аппаратом с программным обеспечением коэффициентов усиления

Летная динамика современного самолета, которая характеризуется высоким уровнем нелинейности и варьируется в широком рабочем диапазоне эксплуатации объекта, усложняет задачу обеспечения необходимых характеристик обратной связи с простым регулятором. Рассмотрен синтез регулятора с программным обеспечением коэффициентов усиления для системы с переменными параметрами с помощью аппарата линейных матричных неравенств. Полученный регулятор гарантирует эффективное управление беспилотным летательным аппаратом под воздействием внешних возмущений в рабочем диапазоне эксплуатации объекта. Процедура синтеза приведена на примере бокового канала беспилотного летательного аппарата.

Ключевые слова: программное обеспечение коэффициентов усиления; линейные матричные неравенства; робастность; беспилотный летательный аппарат.